Quantum State Teleportation understood through the Bohm Interpretation

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Abstract

Quantum state teleportation has focused attention on the role of quantum information. Here we examine quantum teleportation through the Bohm interpretation. This interpretation introduced the notion of active information and we show that it is this information that is exchanged during teleportation. We discuss the relation between our notion of active information and the notion of quantum information introduced by Schumacher.

1 Introduction

The recent discovery of quantum state teleportation [1] has re-focused attention on the nature of quantum information and the role of quantum non-locality in the transfer of information. Developments in this area have involved state interchange teleportation [2], as well as multi-particle entanglement swapping [3], and position/momentum state teleportation [4]. Although these effects arise from a straight forward application of the formalism, the nature of the quantum information and its transfer still presents difficulties. Attempts to address the issue from the perspective of information theory [5],[6] and without invoking wave function collapse [7] have clarified certain aspects of this process but problems still remain.

In order to obtain a different perspective on these phenomena we first review the salient features of the Bohm interpretation that are of direct relevance to these situations [8], [9], [10], [11], before applying its techniques to the specific example of spin teleportation. One of the advantages of using this approach in the present context is that to account for quantum processes it is necessary to introduce of the notion of 'active' information. This notion was introduced by Bohm & Hiley [9] to account for the properties of the quantum potential which cannot be consistently regarded as a *mechanical potential* for reasons explained in Bohm & Hiley [9]. There is also the added advantage that the approach gives a clear physical picture of the process at all times, and, therefore provides an unambiguous description of where and how the 'quantum information' is manifested. In this paper we will discuss how the three notions of active, passive and inactive information are of relevance to the teleportation problem.

2 Quantum Teleportation

The basic structure of quantum teleportation can be expressed using three spin-$\frac{1}{2}$ particles, with particles 2 and 3 initially in a maximally entangled EPRB state, and particle 1, in an unknown superposition:

$$\Psi_1 = (a|\uparrow\rangle_1 + b|\downarrow\rangle_1)(|\uparrow\rangle_2|\downarrow\rangle_3 - |\downarrow\rangle_2|\uparrow\rangle_3)/\sqrt{2}$$

By introducing the 'Bell states'

$$\begin{array}{ll} \beta_1^{(ij)} = (|\uparrow\rangle_i|\uparrow\rangle_j + |\downarrow\rangle_i|\downarrow\rangle_j)/\sqrt{2} & \beta_2^{(ij)} = (|\uparrow\rangle_i|\uparrow\rangle_j - |\downarrow\rangle_i|\downarrow\rangle_j)/\sqrt{2} \\ \beta_3^{(ij)} = (|\uparrow\rangle_i|\downarrow\rangle_j + |\downarrow\rangle_i|\uparrow\rangle_j)/\sqrt{2} & \beta_4^{(ij)} = (|\uparrow\rangle_i|\downarrow\rangle_j - |\downarrow\rangle_i|\uparrow\rangle_j)/\sqrt{2} \end{array}$$

we can re-write Ψ_1 as

$$\begin{aligned}\Psi_2 \;=\; (\beta_1^{(12)} & \;[-b|\uparrow\rangle_3 + a|\downarrow\rangle_3]+ \;\; \beta_2^{(12)} \;[+b|\uparrow\rangle_3 + a|\downarrow\rangle_3]+ \\ \beta_3^{(12)} & \;[-a|\uparrow\rangle_3 + b|\downarrow\rangle_3]+ \;\; \beta_4^{(12)} \;[-a|\uparrow\rangle_3 - b|\downarrow\rangle_3])/2\end{aligned}$$

If we now measure the Bell state of particles 1 and 2, and communicate the result to the recipient of particle 3 who will,using that information, then perform one of the local unitary operations on particle 3 given below

$$\begin{aligned} U_1 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, & U_2 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ U_3 &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, & U_4 &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned}$$

In this way we have disentangled particle 3 from particle 2 and produced the state $(a|\uparrow\rangle_3 + b|\downarrow\rangle_3)$ on particle 3. Thus the information represented by [a,b] has been perfectly 'teleported' from particle 1 to particle 3, without our having measured a or b directly. Furthermore, during the transfer process we have only passed 2 classical bits of information (corresponding only to the choice of U) between the remote particles. Note that as 'a' and 'b' are continuous parameters, it would require an infinite number of classical bits to perfectly specify the [a,b] state. This ability to teleport accurately has been shown to be critically dependant upon the degree of entanglement of particles 2 and 3 [5], [12].

We may note that in the Bell state expansion, the information signified by the coefficients [a,b] appears on the particle 3 spin states before any actual measurement has taken place (although this information is encoded in a different way for each Bell state). What are we to make of this?

It would seem absurd to assume that the information described by a and b was already attached to particle 3 as, at this stage, particle 1 could be any other particle in the universe. Indeed all that has happened is that Ψ_1 has been the re-written in a different basis to give Ψ_2. Clearly this cannot be regarded as an actual physical effect.

Following Heisenberg [13] and Bohm [14], we can regard the wave function as describing potentialities. At this stage Ψ_2 describes the potentiality that particle 3 could carry the [a,b] information that would be actualised during the measurement. However, here we have a problem as Braunstein[7] has shown that a collapse of the wavefunction (the usual mechanism by which such potentialities become actualised) is unnecessary to the description of quantum teleportation, by including the Bell state measuring device within the quantum formalism. Using this descripton, we find that the attachment of the [a,b] information to particle 3, *after* the Bell state interaction, is the same as in the Ψ_2 expansion *prior* to the interaction. While this is clearly necessary to maintain the no-signalling theorem, it leaves ambiguous the question of whether the [a,b] information has been transferred to particle 3, at this stage, or not.

To resolve these issues, we need to give a clearer meaning to the nature of the information contained in [a, b] and to understand how and when this information becomes manifested at particle 3. We now turn to the Bohm interpretation to provide some new insights into these questions.

3 The Quantum Potential as an Information Potential

The Bohm interpretation of quantum mechanics can be derived from the polar decomposition of the wave function of the system, $\Psi = Re^{iS}$, which is inserted into the Schrödinger equation[1]

$$i\frac{\partial\Psi}{\partial t} = \left(-\frac{\nabla^2}{2m} + V\right)\Psi$$

yielding two equations, one that corresponds to the conservation of probability, and the other, a modified Hamilton-Jacobi equation:

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V - \frac{\nabla^2 R}{2mR}$$

[1] We set $\hbar = 1$ throughout this paper.

This equation can be interpreted in the same manner as a classical Hamilton- Jacobi, describing an ensemble of particle trajectories, with momentum $p = \nabla S$, subject to the classical potential V and a new quantum potential $Q = -\frac{\nabla^2 R}{2mR}$. The quantum potential, Q, is responsible for all the non-classical features of the particle. It can be shown that, provided the particle trajectories are distributed with weight R^2 over a set of initial conditions, the weighted distribution of these trajectories will match the statistical results obtained from the usual quantum formalism. It should be noted that although the quantum Hamilton-Jacobi equation can be regarded as a return to a classical deterministic theory, the quantum potential has a number of the non-classical features that make the theory very different from any classical theory. We should regard Q as being a new quality of global energy that augments the kinetic and classical potential energy to ensure the conservation of energy at the quantum level. These non-classical features are of direct relevance to the problem of quantum teleportation. Of particular importance are the properties of non-locality and form-dependence.

3.1 Non-locality

Perhaps the most surprising feature of the Bohm approach is the appearance of non-locality. This feature can be clearly seen when the above equations are generalised to describe more than one particle. In this case the polar decomposition of $\Psi(x_1, x_2, \cdots, x_N) = R(x_1, x_2, \cdots, x_N)e^{iS(x_1, x_2, \cdots, x_N)}$ produces a quantum potential, Q_i, for each particle given by:

$$Q_i = -\frac{\nabla_i^2 R(x_1, x_2, \cdots, x_N)}{2mR(x_1, x_2, \cdots, x_N)}$$

This means that the quantum potential on a given particle i will, in general, depend on the instantaneous positions of all the other particle in the system. Thus an external interaction with one particle may have a non-local effect upon the trajectories of all the other particles in the system. In other words groups of particles in an entangled state are, in this sense, non-separable. In separable states, the overall wave function is a product of individual wave functions

For example, when one of the particles, say particle 1, is separable from the rest, we can write $\Psi(x_1, x_2, \cdots, x_N) = \phi(x_1)\xi(x_2, \cdots, x_N)$. In this case $R(x_1, x_2, \cdots, x_N) = R_1(x_1)R_{2\cdots N}(x_2, \cdots, x_N)$, and therefore:

$$Q_1 = -\frac{\nabla_1^2 R_1(x_1)R_{2\cdots N}(x_2, \cdots, x_N)}{2mR_1(x_1)R_{2\cdots N}(x_2, \cdots, x_N)} = -\frac{\nabla_1^2 R_1(x_1)}{2mR_1(x_1)}$$

In a separable state, the quantum potential does not depend on the position of the other particles in the system. Thus the quantum potential only has non-local effects for entangled states.

3.2 Form dependence

We now want to focus on one feature that led Bohm & Hiley [9] to propose that the quantum potential can be interpreted as an 'information potential'. As we have seen above the quantum potential is derived from the R-field of the solution to the appropriate Schrödinger equation. The R-field is essentially the amplitude of the quantum field Ψ . However, the quantum potential is not dependant upon the amplitude of this field (i.e., the intensity of the R-field), but only upon its *form*. This means that multiplication of R by a constant has no effect upon the value of Q. Thus the quantum potential may have a significant effect upon the motion of a particle even where the value of R is close to zero. One implication of this is that the quantum potential can produce strong effects even for particles separated by a large distance. It is this feature that accounts for the long- range EPRB-type correlation upon which teleportation relies.

It is this form-dependence (amongst others things) that led Bohm & Hiley [9] [16] to suggest that the quantum potential should be interpreted as an information potential, rather than a classical mechanical potential [2] . Here the word 'information' takes its meaning from its etymological roots *informo* (or

[2]It is essential to note that the Bohm interpretation we use in this paper is conceptually different from the "Bohmian mechanics" discussed by Goldstein et al. [21] [For further details see Hiley [22]] Bohm[14] [15] himself rejected the notion of "mechanics" as underlying quantum processes. It was for this reason that Bohm and Hiley[9] were led to introduce the notion of an information potential.

informare) which signifies the *action* of forming or bringing order into something. Thus the proposal is that the quantum potential captures a dynamic, self- organising feature that is at the heart of a quantum process.

For many-body systems, this organisation involves a non-local correlation of the motion of all the bodies in the entangled state, which are all being simultaneously organised by the collective R-field. In this situation they can be said to be drawing upon a common pool of information encoded in the entangled wave function. The informational, rather than mechanical, nature of this potential begins to explain why the quantum potential is not definable in the 3-dimensional physical space of classical potentials but needs a 3N-dimensional configuration space. When one of the particles is in a separable state, that particle will no longer have access to this common pool of information, and will therefore act independently of all the other particles in the group (and vice versa). In this case, the configuration space of the independent particle will be isomorphic to physical space, and its activity will be localised in space-time.

4 Active, Passive and Inactive Information

In order to discuss how and what information is teleported, we must first distinguish between the notions of active, passive and inactive information. All three play a central role in our discussion of teleportation. Where a system is described by a superposition $\Psi(x) = \Psi_a(x) + \Psi_b(x)$, and $\Psi_a(x)$ and $\Psi_b(x)$ are non-overlapping wave-packets, the actual particle position will be located within either one or the other of the wave-packets. The effect of the quantum potential upon the particle trajectory will then depend only upon the form of the wave-packet that contains the particle. We say that the information associated with this wave-packet is active, while it is passive for the other packet. If we bring these wave-packets together, so that they overlap, the previously passive information will become active again, and the recombination will induce complex interference effects on the particle trajectory.

Now let us see how the notion of information accounts for measurement in the Bohm interpretation. Consider a two-body entangled state, such as $\Psi(x_1, x_2) = \phi_a(x_1)\xi_a(x_2) + \phi_b(x_1)\xi_b(x_2)$, where the active information depends upon the simultaneous position of both particle 1 and particle 2. If the ϕ_a and ϕ_b are overlapping wave functions, but the ξ_a and ξ_b are non-overlapping, and the actual position of particle 2 is contained in just one wave-packet, say ξ_a, the active information will be contained only in $\phi_a(x_1)\xi_a(x_2)$, the information in the other branch will be passive. Therefore only the $\phi_a(x_1)$ wave-packet will have an active effect upon the trajectory of particle 1. In other words although ϕ_a and ϕ_b are both non-zero in the vicinity of particle 1, the fact that particle 2 is in $\xi_a(x_2)$ will mean that only $\phi_a(x_1)\xi_a(x_2)$ is active, and thus particle 1 will only be affected by $\phi_a(x_1)$.

If $\phi_a(x_1)$ and $\phi_b(x_1)$ are separated, particle 1 will always be found within the location of $\phi_a(x_1)$. The position of particle 2 may therefore be regarded as providing an accurate measurement of the position of particle 1. Should the ϕ_a and ϕ_b now be brought back to overlap each other, the separation of the wave-packets of particle 2 will continue to ensure that only the information described by $\phi_a(x_1)\xi_a(x_2)$ will be active. To restore activity to the passive branches of the superposition requires that both $\phi_a(x_1)$ and $\phi_b(x_1)$ and $\xi_a(x_2)$ and $\xi_b(x_2)$ be simultaneously brought back into overlapping positions. If the $\xi(x_2)$ represents a thermodynamic, macroscopic device, with many degrees of freedom, and/or interactions with the environment, this will not be realistically possible. If it is never possible to reverse all the processes then the information in the other branch may be said to be inactive (or perhaps better still 'deactivated'), as there is no feasible mechanism by which it may become active again. This process replaces the collapse of the wave function in the usual approach.

5 Quantum State Teleportation and Active Information

In order to examine how these ideas of information can be used in quantum teleportation, we must explain how spin is discussed in the Bohm interpretation. There have been several different approaches to spin [9], [19], [20], but this ambiguity need not concern us here as we are trying to clarify the principles involved. Thus for the purpose of this article we will adopt the simplest model that was introduced by Bohm, Schiller and Tiomno [17], [18]. We start by rewriting the polar decomposition of the wave function as $\Psi = Re^{iS}\Phi$ where Φ is a spinor with unit magnitude and zero average phase. If we write:

$$\Phi = \begin{pmatrix} r_1 e^{is_1} \\ r_2 e^{is_2} \\ \vdots \\ r_n e^{is_n} \end{pmatrix}$$

where n is the dimension of the spinor space, then $\sum_i s_i = 0$ and $\sum_i (r_i)^2 = 1$. The many-body Pauli equation then leads to a modified quantum Hamilton-Jacobi equation given by:

$$\frac{\partial S}{\partial t} - i\Phi^\dagger \frac{\partial \Phi}{\partial t} = -\sum_i \left(\frac{p_i^2}{2m} + Q_i + 2\mu_i \mathbf{B}.\mathbf{s}_i \right)$$

with a momentum $p_i = \nabla_i S + \Phi^\dagger \nabla_i \Phi$, a quantum potential $Q_i = \frac{1}{2m}(-\nabla_i^2 R + \nabla_i \Phi^\dagger \nabla_i \Phi + (\Phi^\dagger \nabla_i \Phi)^2)$. $\mathbf{B}$ is the magnetic field and μ_i is the magnetic dipole moment associated with particle i. We can, in addition, attribute a real physical angular momentum to each particle i given by $\mathbf{s}_i = \frac{1}{2}\Psi^\dagger \sigma_i \Psi$, where σ_i are the Pauli matrices operating solely in the spinor subspace of particle i.

The information contained in the spinor wave function is again encoded in the quantum potential, so that the trajectory of the particle is guided by the evolution of the spinor states, in addition to the classical interaction of the $\mathbf{B}$ field with the magnetic dipole moment of the particle. Contracting the Pauli equation with $\Psi^\dagger \sigma_i$ leads the equation of motion for the particle i spin vector:

$$\frac{d\mathbf{s}_i}{dt} = \mathbf{T}_i + 2\mu_i \mathbf{B} \times \mathbf{s}_i$$

where $\mathbf{T}_i$ is a quantum torque. The k components of the torque are given by

$$[T_i]_k = \sum_j \frac{1}{2\rho m_j} \epsilon_{klm} \{ [s_i]_l [\nabla_j]_n (\rho [\nabla_j]_n [s_i]_m) + s_{lr} [\nabla_j]_n (\rho [\nabla_j]_n s_{mr}) \}$$

where $\rho = R^2$ and s_{ij} is the non-local spin correlation tensor formed from $\Psi^\dagger \sigma_i \sigma_j \Psi$. Equations of motion for these tensors can be derived by contracting the Pauli equation with $\Psi^\dagger \sigma_i \sigma_j$, and similarly for higher dimension correlation tensors. Detailed application of these ideas to the entangled spin state problem has been demonstrated in Dewdney et al. [18].

To complete the description of the particles, we must attach position wave functions to each of the particles. We do this by assuming that each particle can be represented by a localised wave-packet. Thus, for the teleportation problem:

$$\begin{aligned} \Psi &= (a|\uparrow\rangle_1 + b|\downarrow\rangle_1)(|\uparrow\rangle_2|\downarrow\rangle_3 - |\downarrow\rangle_2|\uparrow\rangle_3)\rho(x_1)\phi(x_2)\xi(x_3)/\sqrt{2} \\ &= \{\beta_1^{(12)}[-b|\uparrow\rangle_3 + a|\downarrow\rangle_3] + \beta_2^{(12)}[+b|\uparrow\rangle_3 + a|\downarrow\rangle_3] + \\ &\quad \beta_3^{(12)}[-a|\uparrow\rangle_3 + b|\downarrow\rangle_3] + \beta_4^{(12)}[-a|\uparrow\rangle_3 - b|\downarrow\rangle_3]\}\rho(x_1)\phi(x_2)\xi(x_3)/2 \end{aligned}$$

Initially, the three position wave packets are separable, and the particle trajectories will be determined by separate information potentials although the spin properties of particles 2 and 3 will be linked via the spin quantum potential. The particle spins can be shown to be

$$\mathbf{s}_1 = \frac{1}{2}(a^*b + b^*a, ia^*b - ib^*a, a^*a - b^*b) \quad \mathbf{s}_2 = (0,0,0) \quad \mathbf{s}_3 = (0,0,0)$$

Note that each of the particles 2 and 3 in a maximally entangled anti-symmetric state have zero spin angular momentum, a surprising point that has already been noted and discussed by Dewdney et al. [18] and by Bohm & Hiley [9]. More significantly for our problem is that at this stage, the information described by a and b acts only through the quantum potential, Q_1, which organises the spin of particle 1, but not the spin of particles 2 and 3.

Before discussing the measurement involved in the actual teleportation experiment, let us first recall what happens when a simple spin measurement is made on particle 2 alone. The wave-packet $\phi(x_2)$

would divide into two, and the particle would enter one of these packets with equal probability. Thus the wave function becomes

$$\Psi = (a|\uparrow\rangle_1 + b|\downarrow\rangle_1)\rho(x_1)(|\uparrow\rangle_2|\downarrow\rangle_3\phi_1(x_2) - |\downarrow\rangle_2|\uparrow\rangle_3\phi_0(x_2))\xi(x_3)/2$$

Particle 2 will enter one of the packets, say $\phi_1(x_2)$. As $\phi_1(x_2)$ and $\phi_0(x_2)$ separate, particles 2 and 3 will develop non-zero spins, with opposite senses, and will be described by $|\downarrow\rangle_2|\uparrow\rangle_3$. Any subsequent measurement of the spin of particle 3, would divide $\xi(x_3)$ into two, but particle 3 would always enter the wave-packet on the same branch of the superposition as particle 2 had entered earlier, as only the information in that branch is active. This has been beautifully illustrated by Dewdney et al. [18]

As the particle 1 is in a separable state for both spin and position, no local interactions on particle 2 or 3 will have any effect on the trajectory and spin of particle 1. Neither will any measurement on particle 1 produce any effect on particles 2 and 3. The behaviour of the spins of particles 2 and 3 will be determined by the pool of information common to them both, while only the behaviour of particle 1 is determined by the [a,b] information, regardless of the basis in which the spin states are expanded.

Now let us return to the main theme of this paper and consider the measurement that produces teleportation. Here we need to introduce a Bell state measurement. Let the instrument needed for this measurement be described by the wave-packet $\eta(x_0)$ where x_0 is a variable (or a set of variables) characterising the state of this apparatus. The measurement is achieved via an interaction Hamiltonian that can be written in the form $H = \mathbf{O}^{(12)}\nabla_0$.

The interaction operator $\mathbf{O}^{(12)} = \lambda O_\lambda^{(12)}$ couples the x_0 co-ordinate to the Bell state of particles 1 and 2 through the Bell state projection operators $O_\lambda = \beta_\lambda\beta_\lambda^\dagger$. This creates the state

$$\begin{aligned}\Psi_f \;=\; &\{\eta_1(x_0)\beta_1^{(12)}[-b|\uparrow\rangle_3 + a|\downarrow\rangle_3] + \eta_2(x_0)\beta_2^{(12)}[+b|\uparrow\rangle_3 + a|\downarrow\rangle_3] + \\ &\eta_3(x_0)\beta_3^{(12)}[-a|\uparrow\rangle_3 + b|\downarrow\rangle_3] + \eta_4(x_0)\beta_4^{(12)}[-a|\uparrow\rangle_3 - b|\downarrow\rangle_3]\} \\ &\rho(x_1)\phi(x_2)\xi(x_3)/2\end{aligned}$$

where $\eta_1(x_0)$, $\eta_2(x_0)$, $\eta_3(x_0)$ and $\eta_4(x_0)$ are the wave-packets of the four non-overlapping position states corresponding to the four outcomes of the Bell state measuring instrument. Initially all four systems become entangled and their behaviour will be determined by the new common pool of information. This includes the [a,b] information that was initially associated *only* with particle 1.

As the position variable x_0 of the measuring device enters one of the non- overlapping wave-packets $\eta_i(x_0)$, only one of the branches of the superposition remains active, and the information in the other branches will become passive. As this happens, particle 3 will develop a non-zero particle spin $\mathbf{s_3}$, through the action of the quantum torque. The explicit non-locality of this allows the affects of the Bell state measurement to instantaneously have an effect upon the behaviour of particle 3. The significance of the Ψ_2 Bell state expansion is now revealed as simply the appropriate basis for which the [a,b] information will be transferred entirely onto the behaviour of particle 3, if only a single branch of the superposition were to remain active. The interaction with the Bell state measuring device is required to bring about this change from active to passive information in the other branches (and thereby actualising the potentiality of the remaining branch).

However, no meaningful information on [a,b] may yet be uncovered at particle 3 until it is known which branch is active, as the average over all branches, occuring in an ensemble, will be statistically indistinguisable from no Bell state measurement having taken place. Simply by noting the actual position (x_0) of the measuring device, the observer, near particles 1 and 2, immediately knows which wavepacket x_0 has entered, and therefore which state is active for particle 3. The observer then sends this classical information to the observer at 3 who will then apply the appropriate unitary transformation $U_1 \cdots U_4$ so that the initial spin state of particle 1 can be recovered at particle 3.

6 Conclusion

In the approach we have adopted here, the notion of active information introduced by Bohm and Hiley [9] has been applied to the phenomenon of state teleportation. This gives rise to a different perspective on this phenomenon and provides further insight into the notion of quantum information. To see more clearly how teleportation arises in this approach let us re-examine the above spin example in more general terms. The essential features can be seen by examining the general structure of the quantum potential. Using the initial wave function, Ψ_i given above, the quantum potential takes the form

$$Q(x_1, x_2, x_3) = Q_1(x_1, a, b)Q_{23}(x_2, x_3)$$

Here the coefficients a and b characterise the quantum potential acting only on particle 1. This means that initially the information carried by the pair [a, b] actively operates on particle 1 alone. At this stage the behaviour of particle 3 is independent of a and b, as we would expect.

To perform a Bell State measurement we must couple particle 1 to particle 2 by introducing the interaction Hamiltonian given above. During this process, a quantum potential will be generated that will couple all three particles with the measuring apparatus. When the interaction is over, the final wave function becomes Ψ_f. This will produce a quantum potential that can be written in the form

$$Q(x_1, x_2, x_3, x_0) = Q_{12}(x_1, x_2, x_0)Q_3(x_3, x_0, a, b)$$

Thus after the measurement has been completed, the information contained in a and b has now been encoded in Q_3 which provides the active information for particle 3. Thus we see that the information that was active on particle 1 has been transferred to particle 3. In turn this particle has been decoupled from particle 2. Thus the subsequent spin behaviour of particle 3 will be different after the measurement.

What we see clearly emerging here is that it is *active information* that has been transferred from particle 1 to particle 3 and that this transfer has been mediated by the non-local quantum potential. Let us stress once again that this information is in-formation for the particle and, at this stage has nothing to do with 'information for us'.

Previous discussions involving quantum information have been in terms of its relation to Shannon information theory [23]. In classical information theory, the expression $H(A) = -\sum p_a log_2 p_a$ is regarded as the entropy of the source. Here p_a is the probability that the message source produces the message a. This can be understood to provide a measure of the mean number of bits, per signal, necessary to encode the output of a source. It can also be thought of as a capacity of the source to carry potential information. The interest here is in the transfer of 'information for us'.

Schumacher [23] extended Shannon's ideas to the quantum domain by introducing the notion of a 'qubit' (the number of qubits per quantum system is $log_2(H)$, where H is the dimension of the system Hilbert space). A spin state with two eigenvalues, say 0 and 1, can be used to encode 1 bit of information. To relate this to Shannon's source entropy, Schumacher represents the signal source by a source density operator

$$\rho = \sum_a p(a)\pi_a$$

where $\pi_a = |a_i\rangle\langle a_i|$ is the set of orthogonal operators relevant to the measurements that will be performed and $p(a)$ is the probability of a given eigenvalue being found. The von Neumann information $S(\rho) = Tr(\rho log_2 \rho)$ corresponds to the mean number of qubits, per signal, necessary for efficient transmission. The 'information' in a quantum system, under this definition, is therefore defined only in terms of its belonging to a particular ensemble ρ. It is not possible to speak of the information of the individual system since the von Neumann information of the individual pure state is zero (regardless of the actual values of a and b).

In contrast, in the Bohm interpretation, the information given by [a,b] has an objective significance for each quantum system, it determines the trajectories of the individual particles. The standard interpretation attributes significance only to the quantum state, leaving the particle's position as somewhat ambiguous and, in spite of the appearance of co- ordinate labels in the wave function, there may be a temptation to think that it is the particles themselves that are interchanged under teleportation. This of course is not what happens and the Bohm approach confirms this conclusion, making it quite clear that no particle is teleported. What it also shows is that it is the objective active information contained in the wave function that is transferred from particle 1 to particle 3.

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References

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. K. Wootters, *Phys. Rev. Lett.*, **70**, 1895-99, (1993)

[2] M. H. Y. Moussa, *Phys. Rev.* **A 55**, 3287-90, (1997)

[3] S.Bose, V. Vedral and P. L. Knight, *Phys. Rev.*, **A 56**, 4175-86, (1997)

[4] L.Vaidman, *Phys. Rev.*, **A 49**, 1473-76, (1994)

[5] R. Horodecki and M.Horodecki, *Phys. Rev.*, **A 54**, 1838-43, (1996)

[6] N. J. Cerf and C. Adami, quant-ph/9512022

[7] S. L. Braunstein, *Phys.Rev.*, **A 53**, 1900-02, (1996)

[8] D. Bohm, *Phys. Rev.*, **85**, 166-193, (1952)

[9] D. Bohm & B. J. Hiley, *The Undivided Universe*, Routledge, London (1993)

[10] P. Holland, *The Quantum Theory of Motion*. CUP. (1993)

[11] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*. CUP. (1987)

[12] S. Popescu, *Phys. Rev. Lett.*, **72**, 797-799, (1994)

[13] W. Heisenberg, *Physics and Philosophy*, Harper & Row. (1958)

[14] D. Bohm, *Quantum Theory*, Prentice-Hall. (1951)

[15] D. Bohm, *Causality and Chance in Modern Physics*, Routledge & Paul, (1957)

[16] D. Bohm & B. J. Hiley, *Found. Phys.*, **14**, 255-274 (1984)

[17] D. Bohm, R. Schiller & Tiomno, *Sup Nuovo Cim.*, **1**, 48-66, (1955)

[18] C. Dewdney, P. Holland & Kyprianidis, *J. Phys.*, **A 20**, 4717-4732, (1987)

[19] P. Holland, *Phys. Rep.*, **169**, 293 (1988)

[20] D. Albert, *Quantum Mechanics and Experience* (Chapter 7). HUP. (1992)

[21] D. Durr, S. Goldstein, N. Zanghi, *J.Stat.Phys.*, **67**, 843-907, (1992)

[22] B. J. Hiley, *Active Information and Teleportation* To appear in the Yearbook of the Institute Vienna Circle, (1999).

[23] B. Schumacher, *Phys. Rev.*, **A 51**, 2738-2747, (1995)