PS15

Re-printed from the Proceedings of the International Conference on Elementary Particles 1965 Kyoto, 252-287

Space, Time, and the Quantum Theory Understood in Terms of Discrete Structural Process

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Introduction

In the past half century, there has been a series of revolutionary developments in physics, which includes relativity theory, quantum theory, and the theory, of elementary particles. The effort to bring these theories together has, however, led to a number of fundamental contradictions and ambiguities, which have persistently failed to be resolved. Moreover, recent discoveries in the domain of elementary particles (e g., those that have been interpreted mathematically in terms of groups such as SU(3) suggest that basically new ideas are needed to understand what is going on. So physics is in a state of flux, in which the theories that will eventually emerge may well be as different from current theories as these latter are from those of the nineteenth century.

In this paper, a possible new line of development will be sketched, in which it is suggested that these problems can perhaps be resolved in terms of the notion of space-time as a discrete *structural process.* The meaning of this concept will be brought out gradually throughout the whole article. However, for the present, it will suffice to say that by "structural process", we refer to a set of "space-like" elements which are discrete structures, undergoing discrete or discontinuous changes as they move and unfold in a *process* of development. Such a notion implies that the structural process as a whole, with its set of manifold relationships of partial order of discrete elements, is logically and existentially prior to the notion of a continuous space-time, in the sense that the latter is an abstraction, from the former, representing a kind of approximate "map" of the overall structural process. Such a continuous "map" will have, however, at best a limited domain of validity, and will ultimately have to be dropped in the region of the very small, in favour of going more directly to the discrete structural process from which the " map" has been abstracted.

In some rough sense, the ideas described above mean that both space *and* time ought to be thought of as basically discrete, rather than continuous. But such a terminology is somewhat misleading, because it takes attention away from the essential point that the notion of discrete order relationships that will be described here is very different from any of our usual ideas of space and time, and merely contains the latter as a limiting abstraction. For this reason the notion of discrete order relationship is also very different from that of introducing a "fundamental length." Indeed, it begins by going axiomatically into the notion of order, which is much deeper than that of measure. In every theory of measure or length, order is tacitly assumed and taken for granted, because without knowing what is to be meant by the order of points, the measure of the distance between them, whether this be limited by a fundamental length or not, can have no significance. So this theory is distinguished from others that have been tried thus far, by the fact that it starts by explicating the notion of order, and then comes eventually to measure as a higher level concept. As will be seen, when this is done, one will very probably eventually arrive not at a single "fundamental length", but rather, at a whole series of "fundamental lengths", ordered into a hierarchy of levels.

In a single paper it is, of course, possible only to sketch briefly a few of the main relevant ideas on the subject. Actually, this paper is itself an abstraction from a more extensive work. The object of this work thus far is mainly to clarify some of the tacit assumptions behind our concepts of space and time, and not to predict experimental results. Naturally, however, it is expected that the theory will sooner or later make contact with experiments and a few of the many reasons for such an expectation will be indicated in this paper.

The whole exposition of what follows will be divided into two parts. In the first, we shall go of the notion of what a discrete structural process is, and in the second part, we give a more detailed mathematical theory of this subject.

Part 1. Qualitative Notions of Discrete

Structural Process

1. Analysis of current concepts in physics, suggesting need for a view of space-time as a set of discrete order relationships

We shall begin this part of the work by analyzing some of the current concepts of physics, in order to show that these have in them certain problems and contradictions tending to suggest the need for going to the notion of some kind of discrete structural process that would underlie space and time.

Firstly, let us consider the theory of relativity. The essential new feature of this theory is that there is no absolute simultaneity, so that what is regarded as "present" or "now" depends on the speed of the observer. From this feature, it follows that the geometrical properties of an object (e.g., shape, size and form) do not refer directly to the basic structure of such objects, but rather to certain relatively abstract *relationships* between the object and observing instruments. Thus, if a train is moving, its length is defined as the distance between its end points at the *same time* (e. g., between the rear end today and the front end tomorrow, the distance may be hundreds of miles). And because in relativistic theory different observers do not in general agree on what is the same time, the length (like other geometrical properties) that they attribute to the object cannot correctly be regarded as an "intrinsic" property of that object alone, but rather, it is a *relational property* (as the X coordinate expresses a relationship of a point to an abstract grid).

The revolutionary implications of this feature of relativity are often under emphasised For the "Newtonian" and common sense concept of the world is that it is constituted of a collection of objects, each existing at a given moment, with definite size, shape, and other characteristics. In the next moment, these will all have changed in a certain way. The task of physics is conceived as the analysis of the world into its constituent objects, and the expression of the laws of motion of these objects.

In relativity theory, however, there can be no such objects. Each extended object is actually represented by a *world tube* on a space-time diagram, indicating the motions of all the points within the tube (see Fig. 1). Inside this tube is going on a complex process, involving sub-object, (e. g., a macro-object contains moving atoms which are its sub-objects, while these in turn contain electrons, protons, and neutrons). Ultimately even the sub-objects are seen to be, like the objects themselves, merely abstractions from a total process, built out of suitable sets of ordered series of space-time points representing corresponding movements (these points often being regarded as limiting abstractions of extended events).

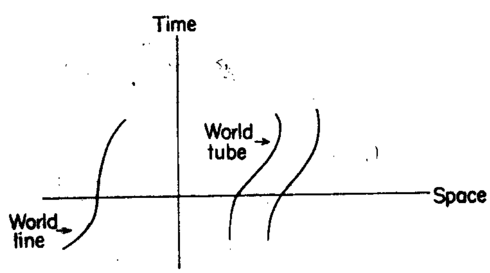

Fig 1.

In the process described above, there is only a *relative permanence* of any particular extended object, a similarity of the general structure and pattern of movement to itself at different times. The concept of the object therefore is no longer a basic one. Rather it is now taken as an abstraction of what is *relatively invariant* in a more fundamental *structural* process involving extended regions of space and time. Thus, the theory of relativity is seen to imply the need to take the concept of a structural process as basic. But this process is of course assumed to be

The attempt to realize a definitive relativity theory of extended structures in the way described above has however always been full of unresolved difficulties. Thus, if there is an extended structure, it is necessary to discuss the interactions of its substructure, which hold them together. This means that we must introduce new and as yet unknown force fields that mediate this interaction. To avoid the ambiguities and confusions that arise from the fact that each substructure must be into yet further substructures, requiring in turn further force fields to mediate their interaction, and so on ad infinitum one may try to abstract from the notion of an extended particle, and treat it as a dimensionless “point", leading to a *world-line* in the space-time diagram, instead of a world tube. But this results in "infinities" in the interaction of a particle with itself, as mediated through the fields that it emits. Thus far, no really consistent way has been found to get rid of such infinities.

Now, let us go on to the quantum theory, which was, as is well known, in many ways even more revolutionary in its implications for or our basic concepts than the theory of relativity. Here, it is necessary to consider only one aspect of the quantum theory, i.e., the existence of discrete quanta of action, and the resulting conclusion that physical processes are not continuous, but are constituted of discrete quantum jumps (e.g., between energy levels of an atom). Although this notion has been confirmed and shown its fruitfulness in a very wide range of applications, it leads to new problems and difficulties, in the attempt to make a relativistic theory of elementary particles. For it is very basic in the theory of relativity that such a particle has to be treated as an extended world tube, with a *continuous structure,* all parts of which undergo *continuous movement.* But according to quantum mechanics, there is no such continuous process. All process is discrete and subject to "quantum fluctuations", not determined by any laws of motion, which become ever bigger as an entity is analysed into finer constituents. It is therefore very unclear as to what could be meant by the extended structure of elementary particles.

If one tries to avoid this problem by treating such particles as extensionless points, one is led not only to the infinities of classical theory, but also to a further set, associated with the quantum fluctuations. To be sure, modern techniques for renormalization provide mathematical algorithms permitting a certain range of effects of interactions to be abstracted from the infinities and to be calculated correctly. But these algorithms have not as yet made possible a deep insight into the nature of the extended structure that such particles must have, if a fully consistent relativistic theory of their interactions with fields is to be developed.

We now consider the evidence that comes from the study of elementary particles themselves. First of all, the notion that these have some kind of extended structure (with a size of the order of 10^{-13} cm to 10^{-14} cm) has been very well confirmed in recent scattering experiments. Besides this, there is further indirect evidence of such structure, in the fact that there exists a whole host of particles, most of them unstable, which can break up, transform into each other, and (in the case of particle -antiparticle pairs) can be created and annihilated. When similar transformability was discovered in atomic nuclei, this indicated clearly that the latter were not "elementary", but have a deeper structure. Similarly, it seems fairly certain that the so called "elementary particles" also have a complex deeper structure, which explains how they can be created, annihilated, and transformed. But the problem still remains as to how such impermanent and extended structures can be treated relativistically and quantum mechanically.

Because of the unresolved problems that have been discussed here, there has developed a school of physics, in which one tries to abandon altogether the motion of an entity with a definite structure, moving in a continuous space-time, and instead one seeks nothing but directly observable relationships, for example, between particles going into a certain region of space (such as a metal target) and those coming out. A few years ago, such a notion of working toward this kind of goal in terms of the " S-matrix " theory was extended by Regge who introduced an interesting mathematical method for treating the particles formally as singularities in the complex planes of the observable variables (such as energy and momentum of the particles in question While this theory is useful in certain ways, it is still subject to a great deal of arbitrariness, as well as to ambiguities and confused questions that arise in the effort to apply it in the relativistic domain. So it has not yet actually indicated any avenue by which the difficulties of making a relativistic theory of elementary particles may be resolved.

2 **On the possibility of developing new principles of order to replace those implied in concept of continuity.**

If one considers the whole line of development of physics which led to the range of problems the expression of the *order and connection* of observable phenomenal as well as the structure of the entities and objects that underlie such phenomena (e. g., atomic structure in a crystal). Now, the evidence described here suggests that there may be something wrong with the assumption that these orders, connections, and structures are basically continuous. But simply to drop space and time altogether (for example, as is done in S-matrix theory) is not enough, because it leaves the situation too arbitrary and confused, with no clear indications as to what are the new orders, connections, and structures that are needed. What is required instead is a new principle of order, connection, and structure, to replace what has thus far been supplied by classical ideas of continuity, which is different from the latter in the small, but which approximates to them in the large.

Our current ideas on continuity came originally from large scale experience. But if one looks carefully, he will see that none of this experience demands such an idea of continuity. Thus, to measure the length of an object, one places each of its endpoints *between* adjacent marks on a ruler, determining in this way a discrete relationship of *order* We translate this for convenience into a *continuous* coordinate, in principle specified to an infinite number of decimals. But evidently, no one measures, observes or otherwise perceives a coordinate to an infinite number of decimals. This is an abstraction, useful in a certain domain, but which has perhaps reached the limits of its usefulness.

How can we go beyond this abstraction? To begin, we can note what are the two basic assumptions underlying the classical notion of continuity. These are:

(a) Between any two points on a line, there is always a third. (This assumption implies the *potential* divisibility of every interval.)

(b) On a small enough scale, whatever is in the interval between a pair of points is not appreciably different from what is at the endpoints This assumption is equivalent to supposing that all physical functions are *continuous* in the sense that their changes in a very small interval can be neglected.

Now, the notion of the *potential* divisibility of any interval is a reasonable one, because the very concept of an interval is such as to enable us inevitably to think of what it would mean to divide it. However, the notion that division ultimately ceases to lead to qualitative change not only falls to follow as a logically necessary consequence of the concept of an interval, but it is also contrary to all the facts that are available. Thus, the division of matter in bulk eventually leads to atoms, which are qualitatively different from matter on the large scale level. Division of atoms leads to electrons, protons, etc., which are in turn qualitatively different from atoms. And now it appears that division of these will lead to something new again at a still deeper level.
More generally, all our experience shows the characteristic that indefinite division of anything is always possible, but that such division ultimately leads to new qualities. For example, when a person is talking, then he feels that there is a relatively continuous flow of words. As an interval is divided, the number of words in each sub-interval is reduced, until one gets down to the interval between successive words. This interval can also be divided, but such a division is meaningless in the level of words. Rather, it involves the experiencing of a qualitatively different mental process (e.g., thinking and understanding what has been said). In other words, the whole of experience is *unbroken,* and yet has a *discrete structure,* which can be analyzed in terms of various levels of ordered intervals, each of which is characterized by its own specific qualities. And, as has been seen, what has been learned in physics shows a tendency to a similar kind of general structure, in which subdivision leads eventually to new qualities and new modes of ordering on different levels.

Now, in terms of classical concepts, one would hope to find a single *fundamental level,* in which everything in the universe could be described in terms of continuous functions of a corresponding set of continuously ordered extensionless points. However, both abstract analysis and factual experience suggest, as we have seen, that the assumption of such an ultimate level is very probably false, as well as self -contradictory in its general implications. Is it possible to avoid such an assumption, and instead to regard the discrete structure of levels indicated previously as basic to what the world is a At this stage, the answer to the question is of course unknown, but it would perhaps be an intelligent guess worth exploring to suppose that our fundamental concept should thus be based on the notion of discrete structure, and of course, discrete process as well.

As a preliminary step toward inquiring into this question, let first emphasize that the ordinary geometrical notion of an extensionless point is not based on any observation or factual knowledge at all. Not only is it impossible, as has been indicated, to measure the coordinates of such a point or to observe it in any other way, but even more, one finds that the very idea of a thinking, while the actual theory is based on introducing the point as a term, which is defined only by the relationships assumed in the axioms (e. g., between points, lines, planes, etc.). And as is well known, these axioms are of ten satisfied in a surprising way by mathematical entities that seem intuitively very different from points. (For example, the set of *lines* in a three dimensional space satisfies the axioms for a set of points in a hypersphere of a six dimensional space.) In our theory we are going then to set aside the notion of an extensionless point and introduce new undefined concepts referring to discrete structural process, with new axioms to relate the new concepts and thus to define their properties by implication.

In developing such a set of new axioms and concepts, it seems reasonable to allow ourselves to be guided by the following requirements.

(a) We must get the usual properties relativistic continuous space time in the large, in suitable approximations.

(b) The deeper level with discrete aspects should give an adequate explanation of the discrete properties of elementary particle (e.g., charge, mass, isospin etc.) along with a treatment of their classifications, structures, and interactions.

(c) One should discover further aspects of this deeper level structure that explain the discrete aspects of quantum mechanics.

(d) Finally, we must come upon something qualitatively new, not contained in present conceptions of physics at all, but which follows from the assumption of a discrete basic structure.

As general relativity led to the qualitatively new feature of relating *metrical properties* of space-time to the physical properties of gravitation and inertia, perhaps the basically new feature of a discrete theory referred to in requirement (d) may be the *order of* the elements. This is simply laid down a priori in a continuous concept *of* space-time, but now it will have to be defined explicitly and related in new ways to physical conception. In terms of such a point of view, our basic observations and measurements will ultimately cease to be directed mainly at revealing the properties of "elementary" particles, but will instead be aimed at answering *new kinds of questions* about the order of basic elements of structural processes, questions that could not even be discussed or framed in terms of current concepts of space-time.

3. **The development of concepts of discrete order, structure and process**

When one wishes to alter concepts as fundamental and far reaching as those of the continuity of structure and order of movement of all matter, he finds that it is necessary to begin with a very general picture covering an extremely broad field of experience in everyday life and in classical physics, even if this has, at first, to be done somewhat hazily. For one has, in *this broad field,* to learn to think in terms of discrete order relationships, rather than in terms of our customary concepts of continuous motion. Indeed, the latter concepts were themselves developed in a long and slow process in which they first appeared in vague, intuitive and tacit forms, only later crystallizing into well-defined abstractions, such as particles and fields, with their energy momentum, mass, charge, etc. If an attempt had been made to *begin* the study of physics by laying down such well defined abstractions, without the preliminary period of working with the "general and somewhat hazy" picture, the whole procedure would have been arbitrary, and would have been most unlikely to lead to anything but a modification of the concepts that were already existent at the time.

Similarly, it would be a mistake to suppose that in this new problem one could skip the initial stage of imprecisely defined generalities in favour of starting from particular pieces of well-defined knowledge, which one hopes to generalize. If one starts from the particular in this way, one is almost certain in fact to be basing his thinking on older *general notions,* containing a host of tacit assumptions that operate almost automatically as unconscious *preconceptions.* As a result, one's intentions to think in a new terms will be contradicted, at the very outset, by the terms that he inevitably uses to describe previously existent pieces of knowledge. From such a point of departure, one will in fact find that on the one hand, the possible directions of generalization seem arbitrary and boundless, while on the other hand, all these directions are seen actually to turn out on closer inspection to be not basically new at all, being merely "variations on a theme" determined by the older set of general tacit assumptions. Moreover, various parts of past knowledge that one wishes to generalize in this way are very likely to come into contradiction, because their real relationships cannot be seen out of the context of the whole broad set of concepts originating in a domain far beyond the limits of a particular field of inevitably be caught in the confusion of trying to mix two sets of ideas that do not actually cohere. In other words, the 14 new wine" of discrete structural concepts should not be poured into the "old bottles" habitually to think of *all* experience, both in physics and more generally.

It is necessary therefore to start, as has been indicated, from a relatively imprecisely defined *general picture*, and then to allow it slowly to "crystallize " In this process, we will be guided by "feeling out " all that is relevant in a wide range of related fields, without jumping in with immediate decisions as to precisely which concepts shall be the basic starting points of further work. The whole new structure of concepts has thus to " precipitate out" slowly and gently, as if from a "solution", which would represent that *unknown* domain of the mind where new kinds of perceptions and ideas can be formed. Each step in such a process is *only provisional,* in the sense that if false starts are made, they will sooner or later reveal themselves as wrong and sources of contradiction, thereafter to be dropped. Then, as the new concepts begin to grow, they will form in a naturally ordered way, rather as happens with a crystal that grows very slowly and gradually into a single coherent structure. So the new ideas will tend to be defined more or less uniquely in consistent and ordered structure that is free of contradiction, with all the parts a naturally integrated and coherent whole.

It must be emphasized that to explore the unknown in this way does not mean that we work solely in the domain of foggy generalities. Rather, it means that a great deal is to be learned by thinking over common and familiar kinds of experience in terms of new concepts. As indicated before, such a process has in fact already occurred long ago, in the development of the foundations of modern physics. To expand a little on this point, it should be noted that when the atomic theory was first proposed, it was not very clear precisely how it should be formulated or how it could be used. What had to be done first was to clarify the atomic concepts to some extent. Later, it became possible to suggest more precise questions both experimental and theoretical (e.g., the statistical explanation of the properties of large scale systems helped to define the radius and the shape of the atoms roughly and vaguely, while only after much more development was it possible to raise precisely defined questions about the properties of individual atoms). What is being suggested here is that the time seems to have come for starting a similar process once again, instead of merely going on with extensions and modifications of existing concepts. Then, when we have learned how a broad range of ordinary and familiar things can be comprehended in terms of new concepts of discrete order and structure, we will perhaps be ready to turn our attention more intelligently to the relatively technical kinds of information made available by recent experimental and theoretical developments in physics, and perhaps eventually to raise entirely new kinds of relatively precise questions that could fruitfully be inquired into.

With this general background in mind, let us now go into the question of developing new concepts of discrete order, structure, and process. As suggested above, we shall begin by considering the problem in its most general possible form, and then gradually make our ideas more precise in such a way as to assimilate the facts of physics, first at the classical level, and later at deeper levels.

Now, if one were asked to give a complete verbal definition of concepts as fundamental as those of order, structure, and process, he will find that this is impossible, because some of the meanings of these concepts are tacitly presupposed in every word and sentence that one can use. Indeed, everyone must already have some kind of set of intuitive mental notions and schemata, which constitute what these concepts really mean to him. So the best that can be done is to start *explicating* what everybody already implicitly knows about order, structure, and process, and then going on to make these common notions more precise, in a mathematical way.

We shall begin then by discussing how the concepts of order and structure are interrelated in all that we observe or experience. Thus, if one wishes to build a *structure* (such as a house) one must in general, start with similar elements (such as bricks) which are, however, also *different* in certain ways (such as the different location, and orientations of different bricks). These differences must be suitably *ordered* or else there will be no structure, but instead, only an arbitrary aggregate of elements.

In the case of the house, the similar bricks are ordered regularly in a two dimensional array to make walls. But in turn, each of these walls can also be regarded as a basic element of a higher order; and then different walls (in different places) can be ordered regularly to make a house. Likewise, houses regarded as similar elements in different locations can be ordered to make streets, streets to make cities, etc., etc. So the overall structure of a city is based on a set of many related orders on different levels. Since each order (e.g., that of the bricks) is the

It seems evident that all structures are built on the principle of order or orders described above. Thus, the cells of the body are based on the ordering of amino acids into higher level elements, such as DNA and RNA molecules, sites for synthesis of proteins, membranes, etc., etc. In turn, each cell is an element of the next level of order, and so on till we reach the body as a whole. A similar principle holds in building the atomic-nucleus out of nucleons, atoms out of electrons and nuclei, molecules out of atoms, galaxies out of stars, etc.

Now, we may try to get a more precise notion of what is *order. To* begin with, one may say that order is based on a set of *similar differences* leading to *different similarities.* Consider, for example, a geometric curve, which is, in some way, an *ordered* set of points. One may analyze such a curve as a set of elements, that could be taken as small lines of equal length, which approximate elements of are, as shown in Fig. 2.

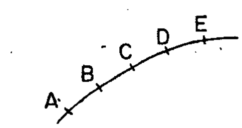

Fig 2.

Each of the line elements is not only similar to the others (e.g., in length) but also different (e.g., in location and orientation). To get a *regular* curve, one must evidently have similar differences. The simplest of these gives rise to the straight line, in which each line differs from the one that comes before in only one way, i.e., that its beginning point coincides with the endpoint of the preceding one, while its direction is the same. In a circle the similar difference is more complex because each line also differs in direction from the preceding one by a constant angle, while it remains in the same plane.

More complex curves can be generated by causing some of the similarities to *differ.* Thus, if the *similarity of* the differences of successive elements is that they all have the same length and the same angle between them, one can also allow this similarity to have the *difference* that the planes of successive pairs of lines are rotated through a constant angle. This will give rise to a simple spiral. And by going on to higher levels of similar differences and different similarities, one can construct more and more complex curves, the complexity of which is reflected in the number of levels of similar differences and different similarities; leading to a *naturally ordered* series of elements. The complexity of such an order is as just objective a feature of it as is the length, and even something that is commonly said to be "chaotic" or "disordered" (e. g., a quasi-ergodic curve) is actually only an extremely complex order of a certain kind.

The paradigmatic case of order in mathematics is that of the integers. Indeed, the integers are similar in being whole numbers, and different in the value that these numbers have. But the differences of successive integers are not only *similar,* they are the *same.* Thus, 2 - 1 = 1, 3 - 2 = 1 4 - 3 = 1, etc. Because of the basic simple relationship that defines their order, the integers provide a natural *mathematical map* of all kinds of order and therefore of structure which is, as has been seen, an order of orders. Thus, to specify the plan of house in such a way that it might in principle be built automatically from a computer programme, one could give a large number of integers, determining by implication how each element is ordered in relation to others, and how the order of orders is built up. Similarly, it will be found that such sets of integers play a key role in providing a mathematical reflection of physical order, not only in molecules, crystals and other such structures, but also in the domain of elementary particles and quantum theory.

One of the basic orders that can be mapped into that of the integers is that of inside and outside Suppose, for example, that we have a moving generator of radiowave pulses that succeed each other at regular intervals (see Fig. 3). Then because light waves are known never to overtake each other, independently of the speed of the source, it follows that at a given moment, a wave arising from a later pulse is always inside the wave from an earlier one. We can thus order the pulses so that the *ordinal number* of each pulse is equal to the total number of the set (i. e. the cardinal number) of pulses that are inside of it. In this way, we represent the basic order of the set of pulses as a set of similar differences that is also similar to the order of the integers. Likewise, it is evident that the time order of pulses is in a one-one correspondence with the order of inside and outside, and therefore also with that of the integers. We see that the principle of orders as similar differences and different similarities is contained in some of the most basic theories, but also in all the experiments with the aid of which the facts that test these theories are obtained.

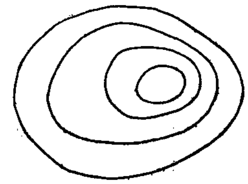

Fig. 3.

Having discussed the terms, "order" and "structure", let us now go on briefly to consider the meaning of the word "process". This too is very familiar to us. Thus, there is the regular process in which a freely moving object changes its position and velocity, the regular way in which it changes its shape under stress and changes its physical state (e.g., solid to liquid) under changes of temperature, etc. In biology, there is the regular process in which the seed or germ cell grows and unfolds to become a plant, which flowers, produces more seeds and dies ' thus playing its part in the life-cycle of the specie of plants, which in turn is evolving on a longer scale of time into new species, etc.

From these and other examples, it is clear that process is in general an *ordered* series of developments of *structure.* Each kind of process is *abstracted* (i.e., taken out in a purely conceptual sense) from its total context, when one makes a *fundamental structural assumption* about such a process. This assumption asserts a range of similar but different structures that are to be regarded as basic in the analysis of processes of this kind. But the key factor which gives rise to an *orderly process* instead of an arbitrary succession of changes, is that one must correctly abstract the *similar differences* in various phases of development, leading on the next level to the proper set of *different similarities.* Therefore, (as happened with the geometric curve), the orders of various phases of a process will follow from some basic and relatively simple *law.*

For example, in the case of a freely moving particle, the basic similar difference is that in successive equal intervals of time, the similarly directed elements of the trajectories *differ* only in a constant space displacement, thus giving rise to one of Newton's laws of motion. If the particle is not free, then the basic similar differences and different similarities are more complex, giving rise generally to what is called in mechanics, the group of "contact transformations". In biology, it is evident that as a plant grows, the differences (e.g., in size at successive intervals of time) are similar, giving rise to the exponential law of growth. But the similarities are in turn different, so that the exponential law does not go on forever, nor does it go on at the same rate in different parts of the plant. Thus, by an ordered differential development of the parts, the plant unfolds its structure and reaches a limit of development, after which it dies. It is easy to see that in other fields, a corresponding analysis of a process of lawful development in terms of similar differences and different similarities is indeed a natural one.

Of course, all basic structural assumptions, along with the assumptions of basic similar differences and different similarities are only provisional. If the development implied by these assumptions is not discovered in factual observation, they can be altered, extended or dropped in favour of very different assumptions. In this way, it becomes clear that scientific research is itself an ordered process. For as the scientist sees how successive phases of development of a theory differ, he can abstract what he regards as the essential difference, which should be similar in the next step, thus suggesting the general direction in which to look for a new idea. However, the similarity must also in some ways be different from what it was before, thus offering scope for genuine novelty and creation.

§4. **On the abstraction of the functional role of structure**

It seems clear then that the notion of a *structural process, as* sketched in the previous section, is a universally applicable one, that is relevant not only in practical life and in scientific research, but also in all of our perception feeling, thinking, and action. What is being suggested here is that it will probably be fruitful (for reasons discussed in §2) to start with a very general notion of this kind as the fundamental one in physics rather than to start with more specialized structural process, and largely ignore some of its more subtle features, which are significant mainly in broader fields. One of the principal features thus emphasized in physics is *function,* which is a certain key aspect of all process.

This notion can properly be applied only under the assumption that both the entire set of structures under discussion, and the set of orders of movement and development by which they are related lie within a fixed and definable domain of possibilities. Thus, it leaves out of account a broader range of process, in which either the basic structural possibilities or the basic order relationships of movement and development as both are fundamentally altered (e.g., a species of animals may evolve to another order of complexity of structural integration, in which some aspects of its basic cellular structure such as the DNA molecules are correspondingly altered). This opening up of new possibilities may be termed a process. Physics however has thus far aimed at avoiding the need to consider such processes, usually with the aid of the tacit assumption that at some fundamental level, perhaps yet to be discovered, the basic structural "elements" and their possible modes of movement and development are limited to certain kinds of order, so that, as it were, the " rules of the game " are at least in principle completely specifiable. Thereafter *everything* could in principle be deduced from these basic laws of nature, not only in the field of physics, but also in every other field of life as well.

It is evidently however not necessary to make such a restrictive assumption Rather, in terms of the point of view that is being suggested here, it seems natural to question the notion that there are "absolute and final truths", such as are implied in the belief that someday, the complete set of all the laws of nature could be known. In fact, the past history of the development of physics indicates that such a notion of absolute truth is in very poor correspondence with what has actually happened. For physics (along with other sciences) has always shown, and is still showing, an unending series of new developments, with no indication whatsoever of any approach to a final and absolutely comprehensive order of natural law.

Indeed, even if we did manage to come into possession of such an absolutely true kind of law, we could not ' in the nature of the case, ever *know* that it is absolutely true. Therefore, no matter what opinion a particular scientist may have about what constitutes absolute truth, he cannot actually do scientific research properly unless he follows the tacit "working hypothesis" that *whatever* we know must in practice be treated as if it had a relative and limited *domain of validity.* Otherwise, it would make no sense, for example, to keep on testing laws that had already been well verified. To express a *verbal* belief in absolute truth while one at the same time is ready in principle to question *every* particular example of a theory that would claim to be such an absolute truth implies however a deeply self -contradictory approach to the whole of scientific research, in which, in effect, a person "says one thing and does another". It would seem to be useful if one's words were to correspond to his actions, at least to the extent of noting that as far as can now be seen, all scientific laws must be treated as *general relationships holding on some limited domain,* the extent of which can be delineated only with the aid of later experimental and theoretical discoveries. A further assertion about the absolute truth of any particular law adds nothing of value to what we already know about that law, and tends merely to create confusion because it engenders a frame of mind in which the law will not be questioned, even when there is evidence that could in principle show that it is false.

If one reflects on the question of what is to be meant by the truth of theoretical ideas, however, one sees that the notion of limited domain of validity of such ideas is indeed a natural consequence of the very way in which we observe and think about the results of our observations. First of all, it is clear that the totality of what we perceive and know is so vast a field that it cannot be understood, without some principles that serve to introduce an appropriate *order* into this perception and knowledge. Within this field, everyone, scientists included, can do nothing but abstract certain aspects that he assumes to be essential and relevant to the whole of experience, while he ignores a tremendous range of other aspects that seem relatively inessential and only of a more limited kind of relevance. By the word "abstraction", we refer here not to something that is cloudy, vaporous, insubstantial or unreal, but rather, simply to the process of *conceptually* "taking out" what is structurally basic in a given situation. Everyone, however "hard headed" and "practical" he may be, is inevitably engaged every moment in this process of abstraction, in which he *orders* his perception and thinking, in such a way that out of it emerges an overall understanding of which aspects are primary, which are secondary, which are causes, which are effects, which are essential and which are superficial, etc. etc. In other words, to engage in the right order of abstraction is the essence of all intelligence, both practical and theoretical.

what he perceives is itself an abstraction from an immeasurably greater field that may be called "the unknown". We are aware that such an abstraction is taking place, if only because later *we come to know* some of what was previously unknown, thus showing that our previous knowledge was incomplete, as well as wrong in parts. And by implication, it seems clear that the same is almost certainly true of present knowledge. For one can see in present knowledge no basic reason even to suspect that it is not, like past knowledge, an abstraction from a broader field that is unknown, from which it follows that the same must almost certainly be true of future knowledge as well. Therefore, the notion of absolute truth is seen to be inappropriate because it is based on false structural assumptions about the nature of human knowledge. This latter is in fact constituted of abstractions that have demonstrated their truth in certain limited domains, that will very probably be true in broader domains, but that will generally ultimately be found to be limited in their validity, when the domain is broader still.

Coming down once again to the relatively restricted and narrow context of physics, we now note that in this field of study, it has been found useful purposely to introduce certain limitations in the domain of possible structures taken into account at each particular stage of development of the subject. Such a limitation permits certain very general properties of natural processes (e. g., space, time, energy, momentum, etc.) to be abstracted, as structurally basic or "essentially" in the domain in question, and thus open up a fruitful range of ways of inquiring into the overall order of natural process. Within this range, one can discover "relatively absolute" laws that apply *invariantly* in suitably defined domains, but which do not apply in broader domains. When the fruitful applications of one domain of possible structure have been exhausted, then physics goes on to consider new domains, which continue the older lines of development in certain key ways, and introduce certain key differences that creatively open up for investigation a new order of structural process.

Within this overall context of abstraction from a broader creative process, both in nature itself, and in physics, which abstracts from nature, we are now ready to define the term "function" more precisely. If we abstract some limited domain of possibilities and denote the totality of them by the term "the field of structure under consideration", then a function *in the domain of this field* is a certain specifiable kind of change or development of structure within that field. Thus, we may say that in the field of the structure of a certain kind of organism, a given organ has a specific function. For example, the function of the heart is to take in blood from the veins and to give it out in the arteries at a higher pressure. The function of the lungs is to transfer oxygen from the air to the blood and carbon dioxide in the reverse direction. One of the functions of the brain is to abstract structural information in the nervous impulses from the sense organs, and on the basis of this information, to determine nervous signals that direct outgoing actions. In engineering, we may say that the function of a bridge is to enable vehicles to go from one side of a river to the other, while the function of a motor is to transform electrical energy into mechanical energy. In communications, we may say that a function of speech is to convey information from one person to another.

In a rough sense, one can say that every function in a certain field is carried out by some substance, object, entity, system or element, having a structure that is in the field under consideration. The function itself is then the action of taking a certain domain of possible structures in what is defined as the "input" and changing or transforming these structures., or else constructing new ones, in the range of what may be called the "output" *A properly defined function* has the basic characteristic that to each possible structure in the input domain, there is a corresponding structure in the output range.

Of course, the role of " input " and " output " is a relational one. Thus, what the "output" of one stage of a *chain of functions* may be the "input" of the next stage. Also, every function can in principle be analyzed or divided into suitable sets of series of *ordered* sub-functions, in which the output of each function is the input of other functions. This analysis corresponds roughly to the possibility of subdivision of a region of space. Of course, as in the case of such a region, the subdivided functions will, in general, have *new qualities.* For example, we may regard a television set as a "black box" with a certain function (to transform electrical signals a visible image). But then we can analyze it in terms of its component parts, which carry out chains of ordered "sub-functions", that are qualitatively different from that of the television set as a whole. In turn, each component can be analyzed in terms of finer-grained ordered chains of function, down to the atoms, electrons, etc.

In a particular context, a certain corresponding level of analysis of function is usually adequate. However, as the context is broadened, then we usually find that a deeper level of analysis is called for. This is just as true in the study of the laws of physics as it is in the other has to bring in first the atomic structure, then the electronic and nucleonic structure, and so on. So in every context, there is always a certain level of function which need not be analyzed further, and can be taken as "relatively elementary" for that context, though we leave open the possibility that it will ultimately have to be analyzed further in a finer context. Since physics always abstracts in some limited context, the basic elements of that context can be taken as discrete, and yet in principle sub-divisible in terms of qualitatively new functions. Thus, we justify the approach which begins with some countable set of discrete elements in a physical theory, though we know that beneath and beyond them may well be an inexhaustably deep and non-countlable infinity of further elements, which can, however, correctly be ignored in the level of abstractions under consideration.

It seems clear that the notion of ordered sets of function and sub-function as discussed above implies the need to understand the *ordering principle* through similar differences and different similarities. Indeed, any one set of ordered functions can be taken as the " element " of a *higher order* of function. Thus, we come in a natural way to the notion of the structure *of function,* which complements that of *function of structure* with which we started. The structure of function is itself evidently a basic branch of knowledge (e.g., in physiology, the structural principle of organization of the sub-functions of a living being is as significant as it is in the understanding of electronic equipment).

But functions can change, and in general, they do. Thus, as an organism matures and ages, many of its functions alter significantly. Since these changes are ordered, function thus undergoes a *structural* process, determined by similar differences and different similarities. The cumulative effects of these changes may eventually lead to the radical alteration of the basic material structure underlying these functions (e.g., a larva becomes a moth). And this process extends further to the *coming into being* of new objects and entitles with their unfolding in a process of development, leading eventually to their passing out of being. (For example, the birth, life and death of individual members of a species of living being.) So the field of function includes the *originating* of integrated material structures and their *dissolution.* Nor does this process of origination and dissolution have to be restricted to some pre-assigned order of development (e. g., the life cycle of an organism). For there can also be a genuine evolution in which new orders of functioning emerge to lead to the origination of new orders of material structures. Thus, we come to the possibility of functions that *evolve,* ultimately to transcend the limits of structural process contained in the original definition of the field of function in question.

One sees in the way that the deeper inquiry into the field of function ultimately carries us beyond the original domain of limited functional abstractions and back into the broader field of *creative process,* from which the abstraction has tacitly been made. But as has been indicated earlier, in each phase of its development physics purposely restricts itself to some limited functional domain that is articulated, developed, explored and extended, until its fruitfulness is exhausted. At this point the fact that nature is actually a creative process and not merely a function operating in some limited field of structure reveals itself in the need for scientific theory to undergo a corresponding creative process by making fundamental changes in basic structure assumptions, which opened up a new field of function for exploration and development. What is at first thus a "creative explosion” of new kinds of ideas and concepts, then gradually "cools down" into a movement of "adapting" existing ideas by developing modifications and variations that assimilate further experimental data. At this point, the theory in question may be said to have more or less 11 crystallized" into a mechanical form, restricting itself to a more or less completely defined set of possible basic structures with correspondingly limited basic orders of development. The further assumption that the field is all that there is ill the whole of nature evidently does not come from anything that could be observed factually, but rather, it is the result of a mechanistic attitude or *philosophy. (Thus, while* a theory may at a certain stage become largely *mechanical only a* particular scientist can be said meaningfully to be *mechanistic in* his attitude. But the actual need for a creative process of developing new basic structural assumptions is always indicating that this mechanistic philosophy is false. For all fields of purely mechanical relationships are ultimately seen to be particular *functional abstractions* from broader creative fields that are not thus limited. So while physics tends to limit itself momentarily to "maps" of nature's process that must eventually cease to be fields of creative thinking and thus have to become more or less *mechanical,* the fact that these forms need to be changed fundamentally from time to time reveals that the subject matter of physics, i.e., the overall order of nature's process, is not in fact purely mechanical.

It is fairly easy to see that the mathematical notion of function and *functional* (or function of functions) is a faithful reflection of the general mode of physical functioning described in the previous section. Thus, if we write

$$y = f(x)$$

we first define a domain of variables, x, and then say that to each value of x (which is the "input", there corresponds a certain value of y (which is the "output"). The function f is then just the overall *relationship* that specifies how the "output" corresponds to the "input" for every value of x in the domain in question.

Such a mathematical function can in general represent some physical function, such as, for example, the electric field. Now usually, the electric field at each point in space and in time, tends to be thought of as an entity, that exists there with a certain value. But as has been seen, this concept of an extensionless entity leads to all sorts of contradictions and confused questions that have not yet been resolved in a satisfactory way. The notion of function discussed above implies, however, a very different point of view about the nature of the electric field. Instead of thinking in terms of the "value" of an electric field as a quality "inhering in- some kind of extensionless entity, we think in terms of the "electrical function" of an extended region of space and time. This function is to accelerate and deflect charged particles that pass through the region in question. If the speed and direction of a particle entering this region is taken as the "input" of such an "electrical function", while those of the particle leaving the region are the “output” then what is now called "the average electric field in the region" determines a correspondence between the "input" and the "output" by means of a simple formula that can easily be worked out.

Interestingly enough, the "electrical function" corresponds exactly to the experimental way of defining the value of the field as proportional to the average force exerted by such a field in a charged particle. It must be emphasized, however, that the notion of function is not equivalent to the philosophy of "operationalism", which identifies all physical properties with nothing more than the totality of results of operation carried out by the scientist, when he measures them. In the point of view that we are developing, the concept of "operations carried out by a scientist is not thus taken as structurally basic for the whole of nature. Rather, it is being assumed here that any one aspect of nature (e.g., the electric field) functions in other aspects (e.g., charged particles). Indeed, the functional relationship of such aspects is in general mutual and reciprocal (e.g., charged particles "generate" electromagnetic fields, i.e., they function in these fields to change their overall patterns). In this way, the concept of a " force " exerted by one " entity " on another and of " interaction " between such entitles" is replaced by the notion that the properties of any element in a domain of structure are always specified from the very outset, by stating the totality of its possible functions in all the elements of that domain. In theoretical physics, such a specification is always carried out in terms of suitable mathematical functions that reflect the relationship of the "input" and "output" in the corresponding physical functions.

One can extend the notions described above by noting that more generally, everything functions *in some way in* everything. However, this function is not of a uniform intensity everywhere. Therefore, relative to a certain level of structural abstraction, only certain classes of elements reveal themselves in the corresponding field of function, so that these can be studied in *conceptual isolation* from the infinity of other classes of elements that do not function relative to this particular level of abstraction. Thus, we explain why it is possible to study partial aspects of the totality of all that exists, despite the universal interconnection of all function.

Once having understood the physical meaning of the mathematical notion of function. we see immediately that the functional (or function of functions) expresses the ordering of function, and its organization, in which each order of function becomes or determines the "input" or "output" of a higher order of function. Consider for example a machine that has a *variable function,* i.e., one that can change in response to the "output" of some other function. (A typical case of this is a vacuum tube, where the functional relationship of plate voltage to plate current depends on the grid voltage, which can in turn be determined by the plate voltage " output" of another such vacuum tube.) In this situation we have one function which depends on another. Such a structure can be extended, so that each function is dependent on a great many others, and ultimately on all the others in the field under discussion. Moreover, the whole set of functions in a given field may influence the nature of a function of another order or level. For example, one to change of temperature). And as will be seen later in more detail, the basic role of a similar application of the notion of functional in quantum mechanical field theory is to reflect the order and structure of the many levels of function that define the fundamental properties of material systems in the quantum mechanical domain.

Until now, we have considered *mathematical function* in the role of directly reflecting the order and structure of *Physical functioning.* It is fairly common, however, for the whole field of mathematical function to *reflect in itself.* For example, a function y = f(x) may be regarded as a *mapping* between x and y, such that to each value of x, there corresponds a value of y, so that the range y now provides a *reflection* or *image* of the domain x. In this role, the notion of functional (function of functions), then permits not merely *mathematical entities* (e.g., numbers) to be reflected in each other, but also *mathematical functions* themselves. Thus, writing Z(x) =f(y(x)), one sees that the *function*, *y(x), is* mapped, or reflected into the function Z(x).

By reflecting the whole field of mathematical function into itself, one can engage in "purely" mathematical reasoning, even to the extent of developing new structures of mathematical function, which may in turn often be capable of directly reflecting new kinds of physical functioning. And indeed, even in physics, one may find it necessary to consider the mapping role of mathematical function itself (e.g., mapping of the movements of particles into coordinate frames). But this mapping role will in general have meaning only in a broader context in which *what is mapped* is taken as logically and existentially prior to the process of mapping. And what is mapped is always *active physical functioning,* which (as in the case of the electric field, for example) is directly reflected into mathematics as a relationship of some "input" to some “output". Such direct reflection of active physical functioning may then be subject to a chain of "mapping" into further mathematical functions but evidently unless this chain can at least in principle begin with some active physical functioning and end in another such functioning, the discussion must remain within a purely mathematical domain.

§6. **The function of the universe as a whole and its reflection in perception, thought, and scientific instruments**

It is clear from the discussion thus far, that the entire universe is interrelated in many y ordered fields of function, with relative independence of each field relative to a suitable lead of abstraction, and ultimate interconnectedness in more extensive and deeper levels of abstraction. This total field of function not only refers to the whole of nature, but also to all human beings and all their activities, including, of course, that of scientific research. Thus, the physicist carries out certain functions in the laboratory (observation and experiment), with the aid of instrumental structures based on the general laws of natural function of all matter. These instrumental structures and their experimental functions are chosen and designed to provide a *perceptible reflection of* aspects of the actual structure and function of the universe, which are in themselves not directly perceptible to the unaided senses.

One of the essential aims of a physical -mathematical theory 'Is then to provide a series of mappings that relates certain directly perceptible results of functioning of the instruments with others. But of course, these mappings depend on suitable underlying *basic assumptions* of order and structure, (generally called by the name of "natural law") as to what happens in the stages that intervene between one such directly perceptible result of functioning and another. If the observed relationships of these functionings are as predicted by the theory, this fact is then taken as a confirmation of the underlying basic assumptions of order and structure. If not, then it is necessary to try out new assumptions, sometimes only slightly different from before, and sometimes much more different, until a confirmation is obtained. In this way, we understand how the order and structure of ideas in physical theories come to reflect order and structure in the general field of functioning according to natural law, with the aid of directly perceptible reflections of natural process provided by scientific instruments.

But then, this whole procedure clearly depends ultimately on the fact that the physicist can *become aware* of the behaviour of his instruments through functioning of his senses, and of his brain, which *thinks about* what is thus perceived. If one considers this point carefully, he will see that the *reflective function* of the physicist's instruments is in certain ways an extension of the similar function of his senses, organs and of his thinking process. So to understand what the physicist is doing, we must briefly consider the broader context of reflective function as a whole, before we abstract down to the narrower field of the relationship of the instruments to the universe that is being investigated.

To understand the reflective process that takes place in perception and thinking, it is useful absorbed and partially reflected and transmitted, with some changes of colour. From the mechanical function one can abstract tactile and kinematical information about the object, while from the optical function, one abstracts visual information. In the subsequent work, we shall be concerned mainly with visual information, the possibility of which is based on the fact that all the light rays passing through each point of space possess an order and structure that reflectively corresponds in a certain rather indirect way to the order and structure of the whole environment. This correspondence is revealed when a person's eye is placed at any point, and the light reaching, that point *functions to* produce a change in the state of the retina of the eye, which is related to the intensity of the light in question. Such a change leads in turn to an alteration of the rate at which cells of the optic nerve are firing, that corresponds to the structure of the light, and therefore of the general environment.

The nerves going from the retina to the brain are connected up in such a way that in the cortex of the latter, certain cells fire, not when a given *point on* the retina is illuminated, but rather, when a certain *extended structure is* present in the illumination. Thus, a specific cell may be found to fire in the presence of a dark spot against a bright background, and others are found to fire when there is a bright spot with a dark background. Neither type fires in a uniform field of illumination. The output of this kind of cell then goes on to become the input of cells of the next level. These function in such a way- that when the light pattern activates a set of spots organized to make up a certain short line segment with a given orientation, then a corresponding cell will fire. In the next deeper level of the cortex, there are certain cells that fire when *any* line in a given neighbourhood is present, with a certain orientation.

To each of these various kinds of function considered in a "positive" sense, there is also in general a corresponding negative or "inhibitory" function, such that the cell in question is prevented from firing in the presence of the structure in question. Then, in the next level, the input of which may contain outputs of various kinds of excitatory and inhibiting functions, it is possible to have a kind of "indifference" or lack of response, to structures to which lower level cells do respond, that is very strongly reminiscent of the interference of waves of opposite phase in physics.

Little is known about the further stages of this process, but the *general* behaviour seems to be that a certain structural feature in the environment makes a corresponding set of cells fire. In this way, the brain responds, not to "points" of light, but rather, to structural patterns of light. Or to put it into our language, we can say that through light and the nervous system, the environment operates with a certain function in the brain, to produce a corresponding result or "output", that reflects not the *immediate form, of the* environment, but rather, some of the series of orders out of which the *structure* of the latter is constituted.

At deeper levels, this function evidently becomes *adaptable, so* that it can be altered by previous experience, enabling a person to respond to new structural features. A key point of this adaptation is carried out with the aid of *thought,* which evidently continues the process of abstraction of structure described above to ever more subtle and comprehensive orders. Thought is *similar in its* operation to that of the optical cortex in that the function of various symbols, such as words, images, patterns of feelings, etc., is *to reflect* structure, rather as the firing of a set of nerve cells does in the simple case of a spot of light or a line segment. But it is different in that the function is not only in principle flexible and adaptable without known limits, but also, in that it can be communicated through the use of corresponding perceptible symbols (such as spoken words, images, gestures, etc.) The transformability and communicability of the function of thought reveals itself in the enormous number of developments of human thinking, including for example, science and mathematics themselves, both of which are evidently high orders of reflective function. In particular, the function of mathematical thought in science has generally been to provide a language having a structure more suited to the reflection of the structure of certain aspects or physical reality than is the common language. Thus, thought is capable not only of reflecting the structure of nature, but as has already been indicated in the -previous section in the discussion of the role of mathematical function, it can also reflect its own structure, and in the very process of doing this, its own structure develops, tranforms and evolves.

This last point makes it clear that the whole discussion is still being carried out at a very abstract level. Not only is the universe as a whole *a creative process* from which various *functional abstractions* are being made. But man himself, who is part of the universe, is also a creative process. In particular, his mind is potentially capable of actions, such as that of perception, understanding, the development of new concepts, etc., going immensely beyond the simple functional notion of thought that has been described here. Indeed, it may be said that *as*

Nevertheless as indicated in S3, it is necessary to begin from the broadest level of abstraction within it that is possible, however vague and hazy this may be, and then come down to narrower levels of abstraction. In this process, one is then likely to discover a *natural order,* that is more or less free of the arbitrariness and confusion that arises when one tries to begin from particular pieces of knowledge.

We have reached a point then, where the whole universe including the human being, his nervous system and brain, with its many levels of reflective function, is seen to be within a single unitary process. Therefore, there is no need to revert to an older point of view, in which theand *what was observed* were conceptually separated almost as if they existed in two different kinds of world, one of which would be "subjective" while the other would be "objective". Rather, there is one total field in which all function takes place, whether it be "outside the skin" or "Inside the skin" of a human being, and whether it be "physical" or "mental". This field of function has the property of universal interconnectedness, with relative separability of various aspects, as described earlier.

One of the kinds of interconnection possible is, as has been seen, the *reflective function.* Such a function takes place even in external nature, when for example, the structure of the whole universe is reflected in the light through each point, in the radio waves that come from the most distant galaxies, etc., etc. The senses and brain then constitute functions that reflect even higher level abstractions from this overall structure. One sees in this way that the structure of thought is itself an extension of the whole structure, which is, like everything else, similar in certain key ways, and different in others.

Therefore, there is no "sharp cut or "unbridgeable gulf" between the most abstract levels of the reflective structure of thought and the general property of the universe, that all structures mutually reflect each other. This reflection is carried along by stages from external nature through the senses and into the brain, where it eventually becomes "thought". But fundamentally, one becomes aware of the structure of thought and the structure of an external object such as a tree in what is basically one process of observation. The tendency to separate these two kinds of structures as if they belonged to different worlds that entered awareness in entirely unrelated ways has, over the past few thousand years, led to enormous confusion in human thinking about the question of "mind" and "matter". Far from being separately existent and structurally unrelated, these are internal and external aspects of one overall creative process, *the totality of all that is.*

When we consider the thought of the scientist in particular, we see that it is abstraction, at least in part, from the similar, but different, reflective function of his instruments. Since, in any case, whatever we know is always an abstraction from an immense and immeasurable field beyond it that is "the unknown", we are necessarily always dealing with partial aspects of reality in our thought. As has been earlier, each such aspect demands a corresponding level of abstraction, and relative to this level, finer-grained and more extensive abstractions can correctly be ignored. When it comes to scientific research, we find that the scientist can be ignored in the process of experiment. Thus, for our purposes from now on, we can say that scientific theories of external nature need in general go only as far as the reflective function of the instrument and do not have to include a detailed account of the reflective function of the sense organs and brain of the scientist himself.

Nevertheless, although the *reflective function of* the instrument is not significantly influenced directly by the scientist who observes the, instrument in question, the *thought of the latter is* contained, in certain ways, in the design and structure of the whole experimental function, that is being carried out in the laboratory. Therefore, *what structural features of the universe the instrument will reflect* are determined by the limited structures of ideas that the scientist is able to consider at a given phase of development of the subject. With new ideas, the scientist can utilize new kinds of instruments in new ways, to reveal hitherto unsuspected structural features of the universe.

It must be emphasized that by including the reflective function of the instrument as an inherent aspect of the function of the universe, as a whole, we have gone beyond the usual point of view, in which it is tacitly assumed that the latter can be "observed" by methods that need not be specified explicitly within the structure of the theory. However, quantum mechanics has already shown that the role of the instrument must be considered when one reaches sufficiently deep levels of physical functioning, but it has not as yet been able to suggest any clear notion of how the reflective function of the instrument is actually carried out. In this work, it will be our ultimate purpose to develop a theory that contains an account of the reflective role of the instrument as a special case of the general property that all levels of function mutually reflect

Indeed, as will be seen, the reflective function is far from an incidental or accidental aspect of natural process that just happens to make it possible for us to observe the world. Rather, it is universal and basic to the whole order of nature in the sense that every function in nature's process includes, *among the totality of its actions,* the possibility of reflecting one feature of the universe in another. Thus, the movement of light has a tremendous number of functions in nature (e.g., it provides energy for plants to grow), but among these, one of them is to provide a reflecion of the structure of the whole universe in each region of space and time. Similarly, the scientist's instruments have many functions (e. g., they deflect objects that strike them and are soluble in acids), but one of these functions is to provide a *directly perceptible* reflection of aspects of the order of functioning of nature that would not otherwise be directly perceptible. The study of reflective function is therefore primarily "ontological" in character (i. c., concerned with the basic structure of what things are) and only secondarily "epistemological" (i.e. concerned with how they come to be reflected in our knowledge and thoughts).

§7. **Referential, inferential and intrinsic structure-the various orders of reflective abstraction**

We shall now go into the structure of the reflective function in more detail, in order to clarify the question of what are the main orders of reflective abstraction in thinking and in scientific research.

To do this, we begin by considering a picture, printed for example in a book. Such a picture evidently fulfils a reflective function. With the general background of lighting as the "Input", to give a structure of light as the "output", which it is the picture. This "output" feeds, of course into the eyes and brain of a human being, where the reflective function is carried to its culminating point in the reflection apprehended in thought and consciousness. In addition, it should be noted that the structure depends on some previous *reflective process,* which may include for example, the functioning of a camera, a photographic plate, and printing equipment. In the terminology that is being used here, the reflective function of the picture is a function of the previous reflective process, so that we deal with a function of a function, or a functional of the whole reflective process (and of *the reflected object itself, as* well).

In such a reflective function, it will in general be found that any particular structural aspect has two possible roles, i. e., *referential* and inferential. *To* illustrate these roles, we may again consider the example of a picture in a book, which is printed in the form of a distribution of little dots. These dots then play the role of a simple referential structure. For once the pattern of the dot-structure is given, then the picture itself is determined by further specifying which dots are printed and which are not (a basically similar procedure was used to transmit pictures from Mars by means of electrical impulses). However, when one looks at the picture, he does not usually notice the dots (though he might if he looked with the specific purpose of seeing them). Rather, he abstracts the *inferential structure,* which is, in this case, the set of objects portrayed in the picture.

Of course, the role of referential and inferential structure in such a series of reflective functions can be interchanged. Thus, if one looks with a microscope at a particular dot, it is seen to be in fact a structure of cells of paper, coloured by ink particles. These latter are now the referential structure, while the dots are playing the role of the inferential structure. This feature of interchangeability of referential and inferential roles on different levels is evidently a general feature of reflective function. Basically, it corresponds to the fact that the "elements" of structure are built out of *ordered sets* of "elements" on the next lower level. Therefore, the higher order elements can always be specified by an abstraction that refers them to the lower order elements.

Generally speaking, however, a good referential structure must have the further property that it does not "get in the way" of the inferential structure that is being " reflected " on it. (For example, the dots must be much finer than any significant feature of the 11 inferential" picture.) When the referential structure has this property, then it will be found, in addition, that the basic relationships in the inferential structure are invariant under a wide range of changes of referential structure. (For example, we can print essentially the same picture with a tremendous of different sizes, shapes, and underlying structures of the array of dots.

In physics, it is common to refer the structure and function of material systems *to coordinate frames,* relative to which everything is taken to be an inferential structure, ordered as "coordinated" in relation to the reference frames in question. In fact, such frames are always realized physically through suitable material structures and processes, such as rulers, clocks, frames specified by certain mathematical functions (e.g., those determining a " space grid"). As in the case of ink dots, it is required that the reference frames shall not impose their own structure on the reflection of physical phenomena that they enter into. To this end, it is required that the theory as a whole be *covariant,* i.e., that it involves relationships in the inferential structure that are invariant to a certain wide class of changes in the referential structure. That is to say, the "relativistic" notion of covariance is just a special case of the more general requirement that inferential structures shall not depend significantly on the referential structures that are used to reflect them.

Although there is a considerable arbitrariness of referential structure the further progress of physics generally lends eventually to the discovery of a "natural reference structure" for each kind of inferential structure. Thus, if we regard an organism as the inferential structure in question, the “natural reference structure" is the set of organs that make it up, which are in turn made up of cells, etc., etc. Similarly, the natural reference structure of a house is that of its constituent bricks, while the natural reference structure of a crystal is that of its constituent atoms. Therefore, while physical laws should not depend significantly on *arbitrary* reference structures, they must depend in a very essential way as natural reference structures. In this way, we come to the notion of an *intrinsic structure,* which is to be understood, not by being referred or related to some other structure, but rather, in *itself*. Each element of such an intrinsic structure is built out of a certain order of other elements, and the whole structure is then seen to be constituted of an overall order of such orders on many levels.

Generally speaking, the proper order of inquiry into as yet unknown structure is to begin by reflecting it on one that is already known. This latter must be such, however, as to be able to "accommodate" itself to the structure under investigation, without imposing such of its own features as may be irrelevant in the problem under consideration. By abstracting from the inferential structure and finding *its invariant relationships,* one then comes to the intrinsic structure, in which each feature is reflected within that intrinsic structure, without the need to refer to extraneous structures.

The overall properties of each aspect of this intrinsic structure are then defined, as has been seen, by the function of that aspect in the whole universe. Such an aspect evidently does not exist separately, any more than the boundary of a room exists separately from the room. Indeed, such a boundary is first of all an abstraction, in the sense that the similar intervals of various kinds in the room come to an end at the wall. But this wall is also only a functional boundary, determined by its action of reflecting light and preventing other solid bodies from entering the space occupied by the wall. At the atomic and nuclear levels, this function is much less sharply defined than it is at our level, and indeed for a neutrino the "boundary' is only a vague nebular "haze", hardly noticeable.

The abstract functional character of the general properties of things can be seen clearly in the case of the boundary of the wall. Nevertheless, there is a tendency to regard the "elementary particles" as the basic constituents of the universe, which really do have a separate and substantial existence. In the notion proposed here, however, elementary particles are *structural-functional* aspects of the whole universe, much as happens with the boundary of the room. And more generally, all other entitles and objects, such as atoms, people, planets and galaxies can be viewed within the framework of the notion of relatively permanent and invariant features of the overall field of *structure-function* much like a regular rhythm as a melody that is carried along in a musical theme. Evidently, the notes are not separately and permanently existing entities that "interact" with each other to make the theme. Rather, the notes are the "natural referential structure" in which the theme is carried as an inferential structure, much as in physics, atoms are the natural referential structure on which the crystal is built as an inferential structure.

In terms of this notion, the concept of a permanent object, entity, or substance is always to be considered as a higher level abstraction of what is only relatively permanent in an ordered totality of actions. In other words, we are carrying to its logical conclusion what is implicit in the relativistic point of view, as described in §2, in which no object or entity at all can without contradiction be taken as structurally basic. We are therefore proposing that to *begin* with the notion of the "elementary" particles and their “interactions" is a wrong order of abstraction, similar to beginning for example with that idea that one could go out and buy a "doorway 11 , which could then be " added " to a wall by means of some kind of " cement " that would cause the two to " interact”. The fact is, of course, that doorway is, like a boundary, a structural abstraction, which has no separate and substantial existence. Likewise, we are suggesting here that elementary particles have no separate and substantial existence, any more than " doorways" analogy to a house is however limited, because in the latter, certain structurally basic and relatively permanent *objects*, such as bricks and beams are involved, while in nature's structural process, we are proposing that the basic "element" is a kind of *action* or *function.* The precise nature of these elementary actions will be discussed in more detail in later sections. But for the present, what is relevant is that particles will be *abstracted* as relatively constant and invariant features of the overall order of action. It will be precisely this structural notion that removes the infinities and other paradoxes that are implicit in current theories of the subject, and that are in fact the results of a wrong order of abstraction from nature's structural

§8. **The "all or nothing" function as basic to all structure in physics.**

We have come to the notion that the reflective function of perception, thought, and knowledge is not something that exists separately from and structurally unrelated to the total field of natural function. On the contrary, such a reflective function is an extension of natural process, similar in certain key ways and different in others to the rest of the total field of function from which it has been abstracted. If we wish to find some basic structural feature that is general enough to apply in the whole field of function, it would seem natural then to begin by seeing whether there is not a kind of "element" that is common to all levels of function, both in external nature, and in its reflection in perception, thought, and consciousness, and then to explore the similar differences and different similarities that constitute the orders and structures that can arise on the basis of these "elements".

Now, it has been that a completely general feature of the reflective function of the nervous system, both in the brain and elsewhere, is that nerve cells produce impulses by firing in bursts, with an "all or nothing" response to some structural feature. For example, at a given moment, a given cell in a certain part of the brain will give a positive response in the presence of certain structures and not in the presence of others. Then there is a different type of cell, which fires when this structure is absent and which is "inhibited" or prevented from firing by a certain range of structures. A similar mode of functioning evidently continues in the higher levels of thought. Thus, a certain set of structures may give rise to corresponding words, images, patterns of feeling, ideas or other mental symbol and reflections to which they correspond, while a different set of structures, to which they do not correspond, either fail to evoke the symbols in question or else even "inhibit" or "negate" them. Thus it follows that the elementary structure of logical thought is similar to that of nervous ' impulses, in the key respect that it is based on the possibility for a structure either to evoke or to negate a corresponding symbolic reflection with an "all or nothing " response.

Going on to external nature, we note that there is a similar but different "all or nothing" response at the foundation of the whole known field of natural law, in the discrete action of all quantum processes. As has been remarked earlier the interference properties (or "wave-like aspects") of quantum processes even seem to reveal something like the "negative " response that can " inhibit " what would otherwise be a "positive " response at certain places. In addition, the discrete properties of elementary particles, such as charge, mass, isospin, etc., suggest that there must be some corresponding rhythm in basic natural processes, which explains why they all have these discrete features in common. And here too, the possibility for negative charges to "annihilate" positive charges suggests some pair of functions similar in relationship to excitatory and inhibitory function.

We are thus led to consider the assumption that all functions have the basic property of being constituted out of ordered structures of "elementary functions". These functions are similar in that they give an "all or nothing" response (either "positive" or "negative") to everything in the field in which they operate. They are different in that their "inputs" and "outputs" are of many different orders, and consist of structures existing on many different but related levels.

As an example of such an " all or nothing" operation in the field of reflective function, we may consider once again the pictures printed in a book in the form of an array of little dots. Each place for a dot has two possibilities. Either it is printed or it is not. So for each place, there is a corresponding "all or nothing" function. The total set of such functions evidently determines the reflective function of the picture as a whole.

One of the many ways in which sets of "all or nothing" functions can be related on different levels is that one kind of such functions determines the *order* of another. Consider for example, a segment of a very fine line, printed out of a single ordered row of dots. Beginning with the dot at one end of the line, we can define higher order "all or nothing" functions belonging to that dot seems clear that the totality of these functions determines the order of dots on the line segment. And these "ordering functions" form a functional (or function of a function) of the original "place elements" that determine the line.

We can now go on to the next level of function. Thus, in a printed image of a square, we can start with one of the boundary lines, and consider the order of lines that are parallel to it. This is determined by a set of “all or nothing" functions of the lines, similar to those that determined the order of the points on a line. Thus, as has been suggested in general terms earlier, we are led to orders, and orders of orders, through the notion of many levels of functions of functions. In this way, we analyze the notion of "generating" lines from points, areas from lines, etc., and essentially we can thus build up all sorts of figures, with all their geometrical relationships. What this means is that the *order* that underlies geometry corresponds to a certain structure of "all or nothing" functions, and these latter correspond, as we have seen, to the general structure of all logical thinking. So we are able thus to develop a fundamental relationship between geometry and logic. This relationship will be brought out in more detail in Part 11.

It is important to note, however, that as geometry is always some kind of abstraction from the whole order of nature, so logic is an abstraction from the whole order of thought. Indeed, the notion of a simple logical order of thought has meaning only in a field in which all of the symbols and terms are well defined, in the sense that they respond as 'gall or nothing" functions to the structures in that field. It is well known, however, that in general, a symbol or term may have a certain vagueness or indefiniteness of its meaning. That is to say, what determines whether it will be brought into operation or not can depend not only on elements in the field under discussion, but also, on elements in other fields, which ultimately extend so widely and deeply that the question of accounting for just what determines its evocation becomes essentially g' imponderable". Of course, such a vagueness of terms of thinking is generally necessary especially in the early stages of an inquiry, in order to allow for flexibility, fluidity, and adaptability in thinking. Eventually, however, as one comes to understand the subject better, the terms of thinking begin to "sharpen up" and when they reach the "all or nothing" stage, they become subject to *logical ordering. In* this regard, the thinking process is very similar to the perceptual process, where likewise our discernment of things initially unfamiliar to us is at first vague, and then sharpens up, as we get a better view. In the limiting case of a complete definition of the "all or nothing" response, belonging to some limited and specifiable field of structure, one has the geometrical abstraction from perception of well-defined figures and the corresponding logical abstraction from thought of well-defined concepts. It is toward this limiting kind of abstraction that scientific and mathematical theories tend to move.

Discussion

Moller I just wanted to ask something similar. Do you have some kind of generalized Lorentz transformation, which would connect your differentials with...

Bohm This is another stage in the development. You see that, this is merely by way of framing the problem, which is that we must see the order of the points in the physical theory. But, there are various ways of making Lorentz invariant theories. The one I applied is to use the spinor as the basic geometrical concept rather than the vector. This turns out to be done by considering the light rays as the fundamental structure rather than some of the other things which I have been discussing. And now the interesting thing is that you take a Dirac spinor, say ψ_1 and ψ_2. You consider the conformal group acting on this, ψ_1, and ψ_2. Then you can not only rotate this spinor and turn it in various directions but you can also move it one place to another. For example a spinor like this represents a light ray through the origin and some other spinor will represent light rays elsewhere. And now say if we consider say a thing like $\psi x \varphi$ for two spinors where that vanishes it represents the intersection of two light rays. Since there are four basic spinors they represent four light rays which intersect in a certain order. Generally speaking, not in a plane so that you form a quadrilateral - which is somewhat twisted. And this 1 take to be a basic structure. Now the point is that by considering a discrete structure of light rays and their intersection, you can work out a theory which is in some approximation- the usual one. The question of approximation 1 want to discuss presently. A conformal transformation turns one light ray into another. Therefore these transformation consists merely of a change of the structure of light rays. And now there are two points to consider, one that the theory is essentially Lorentz rays, you see a Lorentz transformation stretches one and compress another. Now if we take an atom and stretch it to the presumed size of the universe, the horizon of the redshift there will be a certain factor. And you will find that the factor is just about the same as what is needed to compress this light ray down to about 10^{-33} cm. This is a natural gravitational length. So you could say that it would be rather odd to expect Lorentz invariance. You see this indicates the limitations of complete Lorentz invariance for this latter would cause some structure to be spread over the whole universe and another to be compressed down to the fundamental length. Now what 1 propose is that the only thing that limits Lorentz invariance is first of all that if you compress something down to the fundamental length it must come to a limit on the possibility of further Lorentz transformation. You might say this is troublesome, but actually it is desirable because that same compression causes the expansion of some other vector so that again you would expect to come to the limit on Lorentz transformation. This would be very far from energies that are presently available. So a point is that the Lorentz invariance is not necessary feature of covariance. It is a further question.

There is a further invariance which is in this theory, which I don't have time to explain- conformal invariance- which is invariance to inversion through some hypersphere and this in turn, turns out to be in fact what is represented by the operation of charge conjugation. So 1 think this makes a rather reasonable picture in the sense that if you say the theory is invariant to inversion, what it means is that the whole structure of the universe out to this horizon of the red shift is contained in each element. Imagine a spherical mirror. You see now the whole universe out to the horizon is reflected in this mirror, and there will be a certain minimum distance inside this reflection which corresponds to the distance of the horizon. Therefore, if you had the inversion invariance in three dimension you would say that the structure which is reflected in here is the result of inverting the whole universe around this sphere, We can make the assumption that this structure must be invariant to reflection in every region and this will give the idea that the inner structure of each region or each point is a reflection of the whole universe, but a different reflection. And the reflection of the outward horizon is a kind of inward horizon, which would have a size 10^{-33} cm. Although I have not developed it this would be a kind of way of understanding the charge conjugation invariance.