Welcher Weg Experiments from the Bohm Perspective.

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Abstract. We re-examine the claim made by Englert, Scully, Süssman and Walther that in certain 'Welcher Weg' (Which Way) interference experiments, the Bohm trajectories behave in such a bizarre and unacceptable way that they must be considered as unreliable and even 'surreal'. We show that this claim cannot be correct and is based on an incorrect use of the Bohm approach.

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INTRODUCTION.

There is a general perception abroad, although not shared by everybody, that there must be something fundamentally wrong with the Bohm interpretation [BI] of quantum mechanics and if pushed to its limits, BI must produce experimental results that are different from those calculated using the standard approach. This sentiment seems to be based on an instinctive rejection of the approach because, in discussing trajectories, it seems to go against the spirit of the uncertainty principle. Furthermore the standard formalism, as usually presented, does not seem to support the notion of a particle trajectory and it appears as if something new is added. But does the uncertainty principle actually rule this out?

Certainly the uncertainty principle rules out the possibility of *measuring* both position and momentum simultaneously. This fact is not in dispute. The failure of experiments designed to achieve such measurements shows clearly that it cannot be done. Furthermore these attempts bring out clearly why we cannot hope that some cunningly designed experiment in the future will enable us to avoid this difficulty which seems an indisputable feature of quantum processes.

The standard view takes the uncertainty principle to mean that a quantum particle does not *possess* simultaneous position and momentum and exists as some nebulous entity that denies any pictorial representation [1]. But how can we be sure this is the right conclusion? After all no experiment can confirm this conclusion. The experiments merely show that the values of all its properties cannot be known to us simultaneously. It is important to realise that the standard view is based on an *assumption*, namely, that a quantum particle does not possess simultaneously well defined values of all its properties.

However the opposite assumption is also possible, namely, that a quantum particle *does possess* all its properties simultaneously and all the uncertainty principle tells us is that we cannot hope to know the values of all these properties simultaneously. This is exactly what the Bohm interpretation does [2] [3]; it makes the assumption and then explores the consequences that follow from this assumption. There is no ideology involved, just a re-examination of the quantum formalism with this assumption in mind.

The first question one can raise is, "Is it possible to use the present formalism using this assumption without adding any new mathematical content to the theory?" What Bohm [4] showed was that this is indeed possible and that one can provide a logically consistent interpretation based on particle trajectories. Notice it is not being claimed that quantum particles actually follow trajectories, merely that if one assumes they do, then it is possible to produce a consistent account of quantum phenomena. The bonus is that the account is much less puzzling than the standard account with all its paradoxes.

How then does the BI achieve what John Bell [5] called the 'impossible'? Actually it is very simple, so simple that I first thought there must be a catch lurking around the corner. All that one needs to do is to look at the real part of the Schrödinger equation under the polar decomposition of the wave function $\psi(\boldsymbol{r},t) = R(\boldsymbol{r},t)\exp[iS(\boldsymbol{r},t)]$ to find

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$$\frac{\partial S(\boldsymbol{r},t)}{\partial t}+\frac{(\nabla S(\boldsymbol{r},t))^2}{2m}+Q(\boldsymbol{r},t)+V(\boldsymbol{r},t)=0 \tag{1}$$

where $S(\boldsymbol{r},t)$ is the phase of the wave function, $V(\boldsymbol{r},t)$ the classical potential and $Q(\boldsymbol{r},t)=-\nabla^2 R(\boldsymbol{r},t)/2mR(\boldsymbol{r},t)$, where $R(\boldsymbol{r},t)$ is the amplitude of the wave function. I want to emphasise that this result emerges from the Schrödinger equation without adding anything new to the mathematics as can easily be checked.

The probabilistic character of the interpretation enters because it is not possible to produce a quantum particle with given specific simultaneous values of $\{\boldsymbol{r},\boldsymbol{p}\}$. All we have is a distribution of possible values determined by the initial wave function. Thus the probability of a particle being at some initial position $\boldsymbol{r}_0$ is given by $P(\boldsymbol{r}_0,t_0)=|\psi(\boldsymbol{r}_0,t_0)|^2=R^2(\boldsymbol{r}_0,t_0)$. The imaginary part of the Schrödinger equation gives the conservation of probability which ensures agreement with the standard approach at all future times.

Equation (1) looks like the generalisation of the classical Hamilton-Jacobi equation. Indeed if $Q=0$ and S the classical action we would have exactly the classical H-J equation. In this case we have the canonical relations $\frac{\partial S}{\partial t}=-E$ and $\nabla S=\boldsymbol{p}$, which shows that the H-J equation is just the conservation of energy equation. What Bohm did was to exploit this analogy and assume that, in the quantum case, the momentum of the particle is given by

$$\boldsymbol{p}_B(\boldsymbol{r},t)=\nabla S(\boldsymbol{r},t) \tag{2}$$

The particle is then assumed to be specified by its position $\boldsymbol{r}$ and its momentum $\boldsymbol{p}_B$ giving rise to a well defined phase space in which to plot the behaviour of the quantum particle. We can integrate equation (2) and find expressions for a set of trajectories. We then assume the quantum particle will follow one of these trajectories, the specific trajectory will be determined by the initial position of the particle. Examples of how all this works have been given in Bohm and Hiley [3] and Holland [6].

All the examples discussed in the above two references show that the model provides a logically consistent picture of how particles could behave to produce the observed experimental results. But will the approach always produce trajectories that are 'reasonable' and do not predict some form of 'physically unacceptable' behaviour? Not so claims Englert, Scully, Süssman and Walther [ESSW] [7]. They claim the trajectories central to BI produce such bizarre behaviour that they must be considered 'untrustworthy' and even 'surreal'. Scully [9] goes further and claim that this behaviour is contrary to that expected from standard quantum mechanics.

One should immediately be worried by the veracity of such a claim. The BI uses the standard formalism, not in terms of complex numbers, but in terms of the real and imaginary parts of these self-same complex numbers. It adds no new mathematical structure. How is it possible to produce different results?

But wait a moment. We have introduced an additional assumption, namely, that the momentum possessed by the particle is given by equation (2). That, surely, could account for the difference claimed by Scully? Let us examine this point more closely. First we start by reminding ourselves of the probability current density. In the standard theory this is given by the expression

$$j=\frac{1}{2mi}[\psi^*(\nabla\psi)-(\nabla\psi^*)\psi] \tag{3}$$

And if we write the wave function as $\psi=R\exp(iS)$ we find

$$j=R^2\frac{\nabla S}{m} \tag{4}$$

In other words the Bohm momentum is simply the mass probability current. This means that the Bohm trajectories are simply the probability current streamlines. Standard quantum mechanics identifies the probability current with the flux of particles flowing in a certain direction. All Bohm does is to assume that individual particles stick to the flux lines so if the Bohm trajectories are going to do something bizarre, then the probability currents are going to predict exactly the same bizarre behaviour. There cannot be any other conclusion because the mathematical structure is identical in both cases.

So what about the claims to the contrary of ESSW [7] and others [10] [11]? Unfortunately, as we will show in this paper, their conclusions have been reached by not analysing the BI correctly. Before discussing these matters further let us look at how ESSW in particular reach their conclusions.

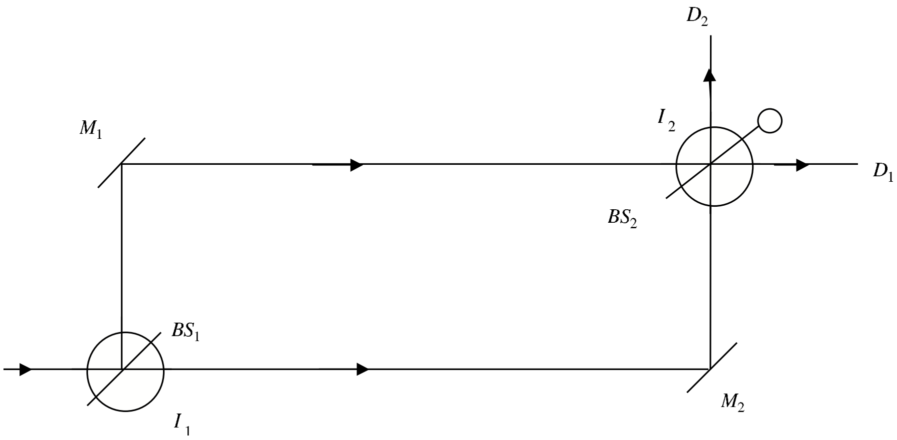

FIGURE 1. Sketch of the Mach-Zender interferometer with beam splitter BS_2 in place. In this case it acts like a wave detector.

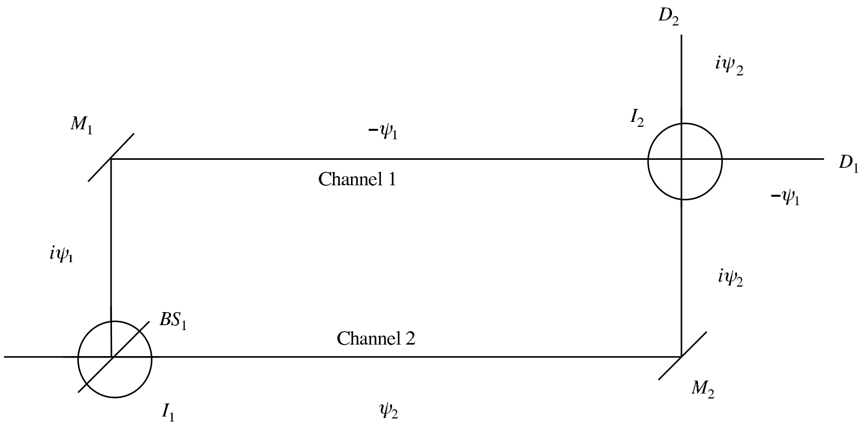

FIGURE 2. Interferometer with BS_2 removed. This makes the interferometer acting like a particle detector.

THE 'WHICH WAY' EXPERIMENTS

The starting point for the criticism is the Mach-Zender interferometer shown in Figure 1. Since the challenge by ESSW involves a Mach-Zender setup using atoms, I will only discuss the interferometer using particles. A discussion entailing photons requires us to generalise the BI to include field theory [12]. This is relatively straight forward and leads to the same conclusion. However the formalism becomes considerably more complicated so there is little to be gained by going into the details in this paper.

As is well known with the beam splitter BS_2 in place the Mach-Zender behaves like a wave detector, because interference between the two paths causes all the particles to end up at the detector D_1. The reason for this is straight forward by standard quantum mechanics. The final wave function at D_1 is the sum of the wave functions travelling down each arm, while the final wave function at D_2 is the difference of these two wave functions. By adjusting the phases and the amplitudes we can arrange the wave function at D_2 to be zero.

With the beam splitter BS_2 removed the interferometer behaves like a particle detector. The final wave function at D_1 is simply the wave function travelling in channel #1, while the final wave function at D_2 is the single wave function travelling down channel #2. (See Figure 2). In this case if BS_1 is 'half-silvered', 50% of the particles arrive at each detector.

Now let us move on to consider how the BI accounts for the particle behaviour in these two forms of the Mach-Zender interferometer. When BS_2 is in place it is easy to see how all the trajectories end up at D_1. There is a fifty-fifty split in the trajectories between channels #1 and #2 at BS_1. When they reach BS_2 the trajectories in channel #1 go straight through BS_2, while those in channel #2 are deflected by BS_2 to reach D_1. These trajectories are sketched in Figure3. One important feature to notice about this diagram is that the trajectories do not cross. This is a consequence

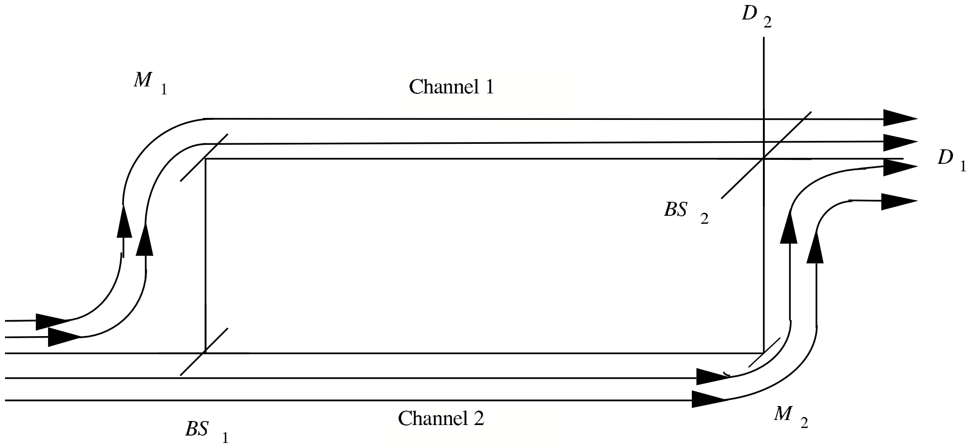

FIGURE 3. Sketch of trajectories with BS_2 in place.

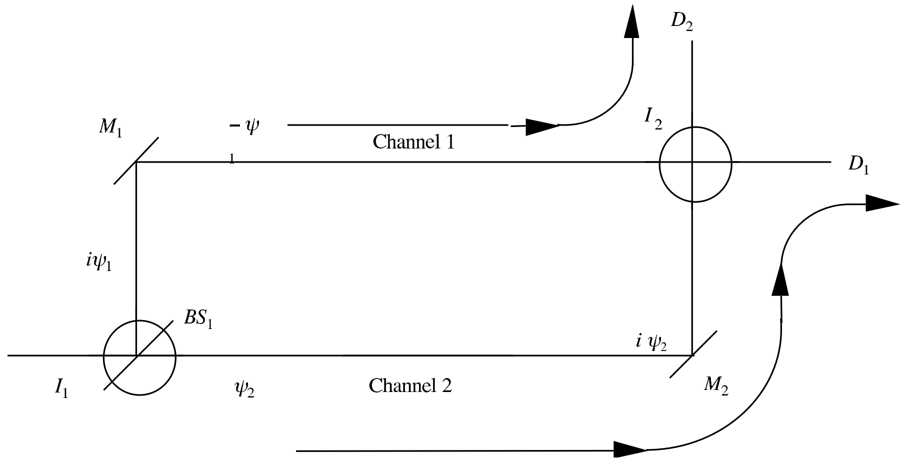

FIGURE 4. Sketch of the Bohm trajectories without BS_2 in place.

of the single valued nature of the field equation (2).

The case when we remove BS_2 is interesting. The trajectories do not cross at BS_2 so that the trajectories travelling along channel #1 bend towards detector D_2 while those following channel #2 bend towards D_1. The behaviour of these trajectories is shown in Figure 4. The reason for this behaviour is that the wave functions in the region I_2 constructively interfere to produce a quantum potential in that region and this potential reflects the trajectories in the appropriate fashion. The details of these calculations will be found in Hiley and Callaghan [13].

There is little to distinguish between the standard approach and that of the BI in these two examples. The real interest and source of the disagreement is when some form of detector is placed in channel #2 to find which way a particular particle will go. This set up is shown in Figure 5. We now go on to discuss this situation.

MACH-ZENDER WITH A CAVITY PLACED IN CHANNEL #2

The detector, in this case a micromaser cavity, is placed in channel #2 and acts as the 'which way' detector. The detector takes the form of a microwave cavity tuned to the internal energy of the atoms used in the experiment. This detector, if correctly tuned, has the property that it will remove the internal energy from an excited atom passing through it without introducing any phase decoherence in the centre of mass momentum of the atom.

One such experiment used Rubidium atoms and the micromaser was tuned to 21.5GHz which corresponded precisely to transitions between the two Rydberg states $63p_{3/2}$ and $61d_{5/2}$. An excellent discussion of the details of the experiment and the theory behind the functioning of the micromaser will be found in ESSW [7] and references

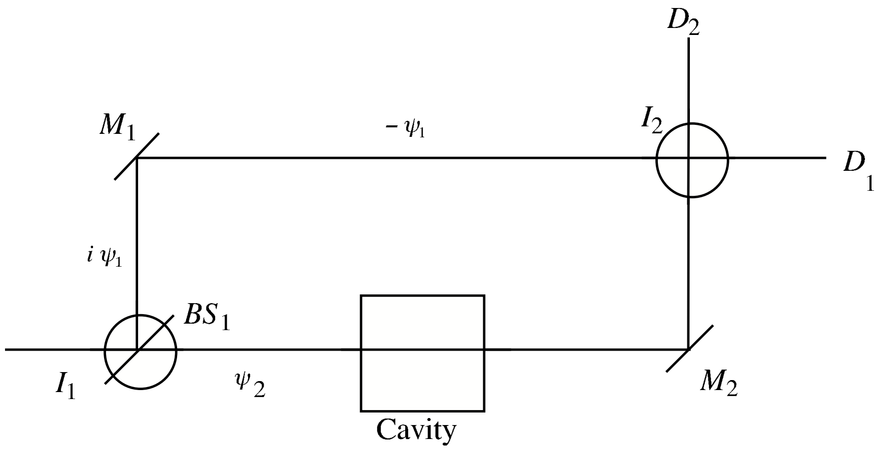

FIGURE 5. Interferometer with cavity in place and BS_2 removed.

therein. The details will not concern us here as they are not in dispute. Here we are interested only in clarifying the principles used by ESSW to arrive at their conclusions about the behaviour of the Bohm trajectories. Again we will simplify the discussion by assuming the micromaser is 100% efficient. This assumption may not be true in an actual experiment but it is not difficult to modify our results to accommodate this feature. It does not change the principle of the argument. This point has been discussed in detail in Hiley and Callaghan [13]

When the cavity is added to channel #2, the essential question concerns the behaviour of the trajectories as they pass through the region I_2 when the beam-splitter BS_2 is removed. Standard quantum mechanics tells us that the particles that arrive at D_1 have not passed through the cavity and therefore still retain their internal energy as can easily be checked by experiment. The particles that pass along channel #2 do lose their internal energy and end up at detector D_2 having lost their internal energy. This result is true whether or not we make a measurement to determine the energy in the micromaser.

All of this is quite straightforward and not controversial. What allegedly causes the problem are the Bohm trajectories. ESSW fix their attention on the fact that Bohm trajectories 'do not cross'. Since this must always hold even in the case when the cavity is added to the interferometer, they conclude that the trajectories must remain the same as sketched in Figure 4 above. If this were a correct picture of the behaviour of the trajectories, it would be truly bizarre. The atoms that travel in channel #1end up at detector D_2. However they have lost their internal energy to the cavity even though the atoms have been nowhere near the cavity. On the other hand the atoms travelling in channel #2 go through the cavity, yet retain their internal energy!

Earlier attempts to explain this result was to claim that this was just another example of the EPR paradox. Dewdney, Hardy and Squires [14], Dürr, Fusseder and Goldstein [15] and even Hiley, Callaghan and Maroney [8] argued in this manner, justifying their conclusion by arguing this is just the true nature of quantum mechanics. Dürr et al [15] remark

> This is somewhat surprising, but if we have learned anything by now about quantum theory, we should have learned to expect surprises.

In spite of such sentiments I was unhappy that such an extreme form of non-locality was required particularly as it involved a non-local transfer of a quantum of energy. Intuitively that felt very wrong, but I could not come up with a convincing argument showing that in *must* be wrong. It was when I was reviewing the situation with Bob Callaghan that it suddenly dawned on me why this could not be correct.

To make this clear it was necessary to recall that the Bohm trajectories were none other than the streamlines of the probability. For me this fact implies that if the trajectories 'don't cross' then the probability current streamlines 'don't cross', a fact of which ESSW were well aware. But if this conclusion is correct then there must necessarily be a contradiction between the predictions using the linearity of the Schrödinger equation and those based on the probability current. Both of these are discussed in terms of standard quantum theory so there is clearly a possibility of a contradiction within quantum theory itself.

TRAJECTORIES CAN 'CROSS'.

To bring out this contradiction, let us replace the 50-50 beam splitter BS_1 with a beam splitter that reflects 75% of the atoms, while allowing the remaining 25% to be transmitted. According to the argument using wave functions, this means that 75% of the atoms reach D_1, while only 25% reach D_2. But according to the probability current, assuming that streamlines don't cross, 75% are deflected into D_2 leaving the rest deflected into D_1. The two results do not add up.

Of course it could be that 50% of the streamlines in channel #1 pass straight through the region I_2 and only 25% of these deflected into D_2. Clearly this can be achieved without the need to have crossing trajectories. But then we have standard quantum mechanics reaching the conclusion that 50% of the atoms that lose their energy to the cavity do not actually go through the cavity! Clearly we have to look much more closely at the behaviour of the streamlines in the region I_2.

The simplest way to do this is to look at the details of the calculation of the trajectories using the BI. Let the wave function of the excited atoms in channel #1 be $\psi_1(\boldsymbol{r})$ and the wave function of the de-excited atoms after they have passed through the cavity be $\psi_2(\boldsymbol{r})$. Let the wave function of the excited cavity be ϕ_e while that of the unexcited cavity be ϕ_{ue}. The wave function in the region I_2 is then

$$\Psi(\boldsymbol{r},\phi) = \psi_1(\boldsymbol{r})\phi_{ue} + \psi_2(\boldsymbol{r})\phi_e \tag{5}$$

The quantum potential acting on the atoms is given by $Q = -\nabla^2_{\boldsymbol{r}} R/2mR$ where R is obtained from

$$\Psi = Re^iS = (R_1 e^{iS_1} R_{ue} e^{iS_{ue}}) + (R_2 e^{iS_2} R_e e^{iS_e}) \tag{6}$$

so that

$$R^2 = (R_1 R_{ue})^2 + (R_2 R_e)^2 + 2R_1 R_2 R_{ue} R_e \cos\Delta S \tag{7}$$

where $\Delta S = (S_1 + S_{ue}) - (S_2 + S_e)$.

Now we come to the crucial part of the discussion. We need to evaluate R for the *actual* trajectory each atom takes. The quantum potential is then calculated at the *actual* position of the atom and the *actual* state of the cavity in each case. Consider an atom following a trajectory in channel #1. Since the interaction with the cavity is local, the atom will not lose its internal energy to the cavity. Thus the probability of the cavity *being* excited must be zero. Remember in BI any system *possesses* values for all its properties. This means that R_e must be zero for this specific atom. Thus each and every atom in channel #1 is acted on by a quantum potential calculated from $R = R_1 R_{ue}$ so there is no phase difference in the region I_2 and no possibility of interference. Thus the atom passes straight through the region I_2 and arrives at the detector D_1 so that the atoms arriving at D_1 will not have lost their energy.

On the other hand when the atom passes through the cavity, it gives up its internal energy to the cavity with 100% efficiency. Thus the probability of the cavity *being* unexcited is zero for this atom, so that R_{ue} must be zero for this atom. Now the quantum potential acting on each atom in channel #2 is calculated from $R = R_2 R_e$. Again there is no interference present and the atoms again go straight through reaching D_2 having lost their internal energy. Thus there is no bizarre behaviour.

We can confirm this result looking at the phase and using $\boldsymbol{p} = \nabla_{\boldsymbol{r}} S$ to calculate the trajectories directly. In the general case when the wave function is given by equation (5), the phase is

$$\tan S = \frac{(R_1 R_{ue})\sin(S_1 + S_{ue}) + (R_2 R_e)\sin(S_2 + S_e)}{(R_1 R_{ue})\cos(S_1 + S_{ue}) + (R_2 R_e)\cos(S_2 + S_e)} \tag{8}$$

Thus for an atom in channel #1 this reduces to $S = S_1 + S_{ue}$ by the arguments used in the paragraph above. This shows that the atoms in channel #1 go straight through I_2 and arrive at detector D_1. It is then obvious that the atoms in channel #2 also go straight through I_2 and arrive at D_2. This confirms that there is no bizarre behaviour predicted on the BI.

We have taken a very simple example here, making a number of simplifying assumptions. However Hiley and Callaghan [13] have considered a variety of different detectors placed in channel #2, going into many detailed questions that we have not considered here. The result is always the same. The behaviour of the atoms is never as predicted by ESSW [7]

CONCLUSIONS.

Our results show that whenever a 'which way' detector is introduced into one of the arms of the Mach-Zender interferometer, the Bohm trajectories pass straight through the region I_2. This implies there is no contradiction of the type suggested by ESSW [7] and Vaidman and Aharonov [10]. The particles that pass through the cavity give up their internal energy and arrive at detector D_1 exactly as the standard approach predicts. To achieve this we do seem to have trajectories that 'cross'. Has this violated the principle that led ESSW to insist that 'trajectories do not cross'? Clearly this is not the case, so how do we understand this situation?

There is one example where we know that trajectories 'cross' and that is in the case where we have a mixed state composed of two incoherent wave function components. In this case it is obvious to see what is going on. There is no interference between the two components of the wave function so they behave independently. Because of this independence the effective configuration space has double the number of dimensions. In this bigger space the trajectories do not cross, so the non-crossing rule still holds but only in the higher dimensional *configuration* space. But in the case that we have been considering above, the two wave functions, $\psi_1(\boldsymbol{r},t)$ and $\psi_2(\boldsymbol{r},t)$ are still coherent and therefore not independent, so why do the trajectories seem to cross?

Again the problem is that we have chosen a configuration space that has too few dimensions. This gives the appearance that the trajectories cross whereas they do not cross in the appropriate space. What distinguishes the two sets of trajectories is the internal energy of the atoms that follow the trajectories. In order to take this difference into account, we must double the dimensionality of the configuration space. In this bigger configuration space the trajectories do not cross. Thus the principle of non-crossing trajectories upon which ESSW based their argument is not wrong *per se*. What was wrong was that ESSW did not consider a big enough configuration space in which to discuss the trajectories. When this fact is recognised then BI gives a completely acceptable account of the experiment. The atoms giving up their internal energy go through the cavity and end up at D_2, while the atoms remaining excited do not go through the cavity and end up at D_1. Thus both standard quantum mechanics and the BI give exactly the same predictions for the outcome of these 'which way' experiments. Thus Scully's conclusion that [9]

> Bohm trajectories are not faithful to the physics of the problem, they are surrealistic.

cannot be sustained. There is no difference between the experimental predictions using the standard formalism and those using the BI.

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