Process and Time*

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Abstract

In this paper we outline an attempt to provide a mathematical description of process from which both space and time can be abstracted. We indicate how spatial properties can be abstracted from the orthogonal Clifford algebra and the symplectic Clifford algebra, algebras that are at the heart of quantum mechanics. The approach is generalised by constructing a bi-algebra from which we are able to abstract a notion of time. The relation of this approach to the work of Prigogine and Umewaza is discussed.

1 Introduction

In a recent paper Haag (1990) has pointed out that even after a collective and sustained agonising over quantum field theory, there still remain a number of dark spots that veil some fundamental internal incompatibility within the conceptual framework of the theory. This unease becomes particularly significant when attempts are made to quantize the gravitational field (See Isham 1987). Some of these concerns have influenced our thinking over recent years and it is for this reason that we have been exploring somewhat different approaches to these questions. One approach has been extremely conservative since it tries to explore the precise differences between quantum and classical phenomena by rewriting the Schrödinger equation in a form that is closer to the Hamilton-Jacobi equation of classical mechanics (Bohm and Hiley 1993), while others have been of a more radical nature (Bohm and Hiley 1984; Frescura and Hiley 1984 and Hiley and Monk 1993).

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The more radical approaches have some features in common with Haag's own proposals, particularly in sharing his general view that we should regard the space-time continuum not as a priori given, but to be a structure derived from something more primitive which contains within it features that are more appropriate to the quantum domain. Our interest in this field was considerably influenced by the Penrose twistor programme (1972) although we were first alerted to more radical possibilities after reading the following three sentences in Eddington's classic work "The Mathematical Theory of Relativity" (Eddington 1937).

> The hiatus (in trying to unite general relativity with electromagnetism) probably indicates something more than a temporary weakness of the rigorous deduction. It means that space and time are only approximate conceptions, which must ultimately give way to a more general conception of ordering of events in nature not expressible in terms of a fourfold co-ordinate system. It is in this direction that some physicists hope to find a solution of the contradictions of the quantum theory.

Clearly, in this view, it is not sufficient merely to strive for frame independent expressions since they assume that the relevant structure of the underlying physical processes can be captured within the very order of the continuum that supports a co-ordinate frame in the first place. As Isham (1987) has argued, it is possible that the order contained in the structure of the differential manifold itself is not appropriate for exploring the structure of space-time near the Planck length and that any alteration to the underlying structure may have profound implications at much larger scales. The proposal is that we should make a thorough re-examination of the underlying assumptions of our theories which should include a re-examination of the topology in the small, with a view to developing more appropriate structures such as "quantum topology" or more generally what Bohm (1965) has called a "quantum topo-chronology". The choice of this word is intended to signify that not only must we emphasise spatial topology i.e., the study of the order of placing one thing in relationship to another, but the study of how one event or moment acts physically in another. In other words topo-chronology is not limited to the concepts of neighbourhood, incidence, boundary and closure, but will include novel notions of relationship, structure and order involving process in general.

This was one of the motivations behind Bohm's (1980) introduction of his novel orders, namely, the implicate and explicate orders, notions that have been criticised as "vague", a criticism implying that these orders can have no value in physics. The intention behind the introduction of these new orders was simply to provide a framework within which to develop new physical theories together with the appropriate mathematical structures that will lead to new insights into the behaviour of matter and ultimately to new experimental tests. This paper is an attempt to develop these arguments further and to suggest appropriate ways of proceeding.

As we have already remarked, the idea of topo-chronology means that we must study order, not only in the static sense, but in an active sense. Thus nature is to be regarded as a structure or order in an evolving process. This process is not to be regarded as a process evolving in space-time, but space-time is to be regarded as a higher order abstraction arising from this process involving events and abstracted notions of "space (or space-like) points". These points are active in the sense that each point is a process that preserves its identity and its incidence relations with neighbouring points. In Hiley and Monk (1993) we described how this could be achieved in a very simple algebraic structure, namely, the discrete Weyl algebra. Thus like Haag (1990), our basic structure assumes that an element of process that can be described by an element of some suitable algebra. For example, it could be a Clifford algebra (Hiley and Frescura 1980a), an Einstein algebra (Geroch 1972) or a Hopf algebra (Zai-Zhe Zhong 1995) .

The basic idea of assuming elements of an algebra can represent structure process goes back to the end of the previous century when Grassmann (1894), Hamilton (1967) and Clifford (1882) were laying the foundations of what were to become the Grassmann, the quaternion and the Clifford algebras. We feel that it is no coincidence that these algebras play a fundamental role in our present physical theories and are structures that are valid in both the classical and quantum domains. These early motivations stemmed from an assumption that metaphysics was a legitimate tool for exploring new ideas both in mathematics and physics hence the emphasis that these pioneers placed on process and activity. For example, Hankins (1976) recalls that at the 1835 meeting of the British Association in Dublin, Hamilton read a paper entitled "Theory of Conjugate functions, or Algebraic Couples; with a Preliminary and Elementary Essay on the Algebra as a Science of Pure Time" in which he freely uses metaphysical arguments to develop his ideas. Again in one of his notebooks he writes:

> In all Mathematical Science we consider and compare relations.
> In algebra the relations which we first consider and compare,

> are relations between successive states of some changing thing or thought. And numbers are the names or nouns of algebra; marks or signs, by which one of these successive states may be remembered and distinguished from another.....Relations between successive thoughts thus viewed as successive states of one more general and changing thought, are the primary relations of algebra.....For with Time and Space we connect all continuous change, and by symbols of Time and Space we reason on and realise progression. Our marks of temporal and local site, our then and there, are once signs and instruments by which thoughts become things....

Unfortunately the concept of process is difficult to work with and already by the 1880s the active role of the elements of the algebra had been lost. The elements of the Grassmann algebra had become the static forms of the vector, the bivector, etc., together the exterior product that we now associate with a Grassmann algebra. It was for this reason that Clifford (1882) found it necessary to emphasise once again that the quaternion product was based on movement or activity rather than a static structure as the Grassmann algebra had become. Indeed in developing his algebra, Clifford constantly emphasises the active role of the elements of the algebra, giving them names such as "rotors" and "motors".

We wish to revive the spirit of those early explorations by taking process as fundamental and to construct space-time, fields and matter from this basic process. To do this we will follow Hamilton and assume that process is describable by elements of an algebra, while the relevant structure process is defined by the algebra itself. The relevance of these ideas to quantum phenomena is suggested by the realisation that the quantum formalism is, in essence, an algebraic theory, and the insistence that it is a description that requires Hilbert space is forced on it by the interpretation that the wave function is related to probability. In our view Hilbert space has become overemphasised in physics and this in spite of the fact that in order to accommodate wave functions for position and momentum, we are forced to a larger structure, the rigged Hilbert space. In a much neglected paper, Dirac (1965) argues that in quantum field theory

> ...the Heisenberg picture is a good picture, the Schrödinger picture is a bad picture, and the two pictures are not equivalent (in field theory), as physicists usually suppose.

By staying in the Heisenberg picture, he then goes on to show that it is possible to obtain solutions of physical problems in this picture when no solution is possible in the Schrödinger picture. In this way Dirac concluded that the Heisenberg picture is more general and since it is an algebraic approach to quantum phenomena, it suggests that the algebra may be of more significance. Unfortunately the difficulty that Dirac saw is that we have no interpretation of a purely algebraic approach. The approach we are suggesting here is an attempt to provide such an interpretation and to do this we must develop a structure in which process is taken as basic, opening up the possibility of interpreting the elements of the algebra in terms of this underlying process. We are far from realising such an approach but at least it is possible to make a start as we will now show.

Let us begin with the assumption that underlying quantum phenomena there is a structure that requires us to take the notion of process as the basic form. Rather than getting into a philosophical discussion as to the meaning of the term "process", let us see if a careful examination of the algebraic structure that lies behind the quantum theory can reveal insights into the nature of this underlying process. After all it was quantum mechanics that led to a revival of interest in the Clifford algebra with the success of the Dirac theory. Here the dynamical variables require us to use elements of the Clifford algebra and the wave functions themselves can be expressed in terms of spinors. As has been pointed out by Frescura and Hiley (1980a) and by Benn and Tucker (1983), these spinors arise in the algebra itself as elements of a minimal left ideal, so that it is not necessary to use the Hilbert space formalism in a fundamental way. Likewise the Penrose twistor programme, although it has not reached the same status as the Dirac theory, exploits the spinors of the conformal Clifford algebra which, in turn, contains the Dirac Clifford algebra as a substructure. In fact Bohm and Hiley (1984) have already shown how the algebraic structure contained in the twistor foralism can lead to a novel approach to the notion of pre-space.

What we regard as an additional significant point is that any Clifford algebra can be constructed from a pair of dual Grassmann algebras. The elements of these Grassmann algebras are analogous to the annihilation and creation operators used in the Dirac approach to fermion fields, thus giving us the possibility of a new significance to these operators in terms of the underlying process, rather than merely as the creation and annihilation of excitations of the quantum fields.

Recently we have been exploring an analogous structure, namely, the symplectic Clifford algebra (Fernandes and Hiley 1996) which can be constructed from boson annihilation and creation operators. This algebra contains the Heisenberg algebra, again suggesting that it will strongly feature in a process orientated approach to quantum theory. Indeed it was these possibilities that lead Hiley and Monk (1993) to explore a simpler finite structure, the discrete Weyl algebra that we have already referred to above. What we have recently been made aware of is the possibility of using a pair of dual boson operators to introduce the notion of time, a feature that was lacking in our own approach (Umezawa 1993). We will discuss these ideas toward the end of this paper.

2 The Algebra of Process

Let us start by illustrating the type of approach that we have been following. We have already intimated above that by starting with Grassmann's original motivations, it is possible to generate a Clifford algebra of movements (For more details see Hiley 1994). Let us regard process as a continuous, undivided flux or flow. To be able to discuss such a flow, it is necessary to distinguish certain features within the flux. The key question to ask is: How is one distinctive feature related to another? In a succession of features, is each feature independent of the others or is there some essential dependence of one on the other? The answer to the first part of the question is clearly "no" because each feature is reflected in the others in an inseparable way. Thus one feature can contain the potentialities for a succeeding feature and each feature will contain a trace of its predecessor.

Let us label two distinguished features by P_1 and P_2 , and regard them as the opposite poles of an indivisible process. To emphasise the indivisibility of this process, we will follow Grassmann and write the mathematical expression for this process between a pair of braces as $[P_1P_2]$. Here the braces emphasise that P_1 and P_2 cannot be separated. Note that the order of the elements is significant, implying the order of succession, i.e., $[P_1P_2] = -[P_2P_1]$. This could be construed as already suggests that a primitive notion of time is being introduced. However no notion of a universal time order is implied. When applied to space, these braces were called extensives by Grassmann.

It should be noted that any attempt to discuss and describe order without introducing time is very difficult. For example, it is clear that the order in any map is timeless, nevertheless it is easier to describe its order in terms of a sequence of directions such as "Turn right when you come to the next cross-roads" and so on. Similarly when we come to discuss the order of physical process, we trace out the order in time for convenience, but this does not imply that the process itself evolves in that time order. Thus in discussing the order of distinguished points in our structure process, it is convenient to discuss the order in terms of "succession" even though no time may be involved in the order we are discussing. Our distinguished points are not then a sequence of independent points, but each successive point is the opposite pole of its immediate predecessors so that points become essentially related in an active way.

For more complex structures, we can generalise these basic processes to: $[P_1P_2]$, $[P_1P_2P_3]$, $[P_1P_2P_3P_4]$ etc. In this way we have a field of extensives from which we can construct a multiplex of relations within the overall process or activity. The sum total of all such relations constitutes what Bohm has termed the holomovement. Thus, in this view, space cannot be a static receptacle for matter; it is a dynamic, active structure.

We now argue that process forms an algebra over the real field in the following sense:

(1) multiplication by a real scalar denotes the strength of the process.

(2) The addition of two processes produces a new process. A mechanical analogy of this is the motion that arises when two harmonic oscillations at right angles are combined. It is well-known that these produce an elliptical motion when the phases are adjusted appropriately. This addition of processes can be regarded as an expression of the order of co-existence.

(3) To complete the algebra there is a inner multiplication of processes defined by $[P_1P_2][P_2P_3] = [P_1P_3]$ which will be called the order of "succession" using this word in the sense explained above.

3 The Clifford Algebra of Process

We now illustrate briefly how this algebra can carry the directional properties of space within it without the need to introduce a co-ordinate system. Let us start by assuming there are three basic movements, which corresponds to the fact that space has three dimensions. We leave open the question as to why only three space dimensions are needed. The scheme can be generalised easily to higher dimensional spaces with different metrics.

Let these three movements be $[P_0P_1]$, $[P_0P_2]$, $[P_0P_3]$. To describe movements that take us from $[P_0P_1]$ to $[P_0P_2]$, from $[P_0P_1]$ to $[P_0P_3]$ and from $[P_0P_2]$ to $[P_0P_3]$, we need a set of movements $[P_0P_1P_0P_2]$, $[P_0P_1P_0P_3]$ and $[P_0P_2P_0P_3]$. At this stage the notation is looking a bit clumsy so it will be simplified by writing the six basic movements as [a], [b], [c], [ab], [ac], [bc].

We now use the order of succession to establish

[ab][bc] = [ac]; [ac][cb] = [ab]; [ba][ac] = [bc]

where the rule for the product (contracting) is self evident. There exist in the algebra three two-sided units, [aa], [bb], and [cc]. For simplicity we will replace these elements by the unit element 1. This can be justified by the following results [aa][ab] = [ab] and [ba][aa] = [ba]; [bb][ba] = [ba], etc. So that

$$\begin{aligned} [ab][ab] &= -[ab][ba] = -[aa] = -1 \\ [ac][ac] &= -[ac][ca] = -[cc] = -1 \\ [bc][bc] &= -[bc][cb] = -[bb] = -1 \end{aligned}$$

There is the possibility of forming [abc]. This gives

$$\begin{aligned} [abc][abc] &= -[abc][acb] = [abc][cab] = [ab][ab] = -1; \\ [abc][cb] &= [ab][b] = [a]; \end{aligned}$$

etc. Thus the algebra closes on itself and it is a straight forward exercise to show that it is isomorphic to the Clifford algebra C(2) which was called the Pauli-Clifford algebra in Frescura and Hiley (1980a).

The significance of this algebra is that it carries the rotational symmetries and for this reason it is called the directional calculus. The movements [ab], [ac] and [bc] generate the Clifford group which is homomorphic to the Lie group SO(3), the group of ordinary rotations. For good measure this algebra also contains the spinors as minimal left ideals which form a linear sub-space in the algebra. This relationship shows that the spinors arise naturally in an algebra of movements. An extension of this algebra to the Dirac algebra then enables us to discuss the light cone structure so that the light ray itself can be given an algebraic meaning. The background to all of this has been discussed in Frescura and Hiley (1984).

4 The Algebra of Points

The calculus of directions is based on the existence of a single point around which the directions emanate. In the Dirac algebra this would be equivalent to considering a light cone at a single point. In order to relate light cones at different points, we have to extend these ideas. We then have two possibilities. We can enlarge the algebra to the conformal Clifford algebra and use the corresponding spinors (i.e., the twistors) to relate light cones at different points in space. This is the basic idea that lies at the heart of the twistor programme. An alternative way involves generalising the algebraic structure to include the kinematics. To do this we introduced a generalised phase space algebra containing translations within it. These translations can also be used to relate light cones at different points (Bohm and Hiley 1981). The algebra that contains these translations is the symplectic Clifford algebra (See Crumeyrolle 1990) or at least an extension of it which has been developed by Frescura and Hiley (1984, 1980b) and Fernandes and Hiley (1996).

We do not want to discuss the details of this structure here as they can be found in the above papers, but it should be noted that this algebra contains the Heisenberg algebra as an automorphism algebra. This clearly shows that our overall approach uses another important aspect of quantum theory that is missing from the twistor programme. Furthermore the symplectic Clifford algebra can be generated by bosonic creation and annihilation operators in a manner that is very similar to the way one generates the orthogonal Clifford algebra by a pair of dual Grassmann algebras as mentioned above. However it should be emphasised once again that in our approach we are not talking about the creation and annihilation of particles, but the generation and annihilation of elements in our underlying process. In this sense the underlying structure process can be thought of as being quantum in essence and therefore can be regarded as a kind of quantum geometry from which the classical space-time will emerge by some suitable averaging.

As we have already mentioned these ideas can be illustrated using a "toy" algebra, namely, the discrete Weyl algebra. (See Hiley 1991 and Hiley and Monk 1994). Although this algebra is very limited, it nevertheless gives an indication of how the idea works. Here one can construct the points of space directly in terms of the minimum left ideals of the algebra so that the spinors of the Weyl algebra represent the 'points' of our generalised phase space.

In order to get some feeling of how these general concepts can be given meaning, let us consider some details. The discrete Weyl algebra is a finite polynomial algebra generated by a set of elements $1, e_1^0, e_0^1$ subject to the relations

$$e_0^1 e_1^0 = \omega e_1^0 e_0^1; \qquad (e_0^1)^n = 1; \qquad (e_1^0)^n = 1 \tag{1}$$

where $\omega = exp(2i\pi/n)$.

One of the reasons for choosing this algebra is that in the limit when

$n \to \infty$, we obtain the symplectic Clifford algebra referred to above. The fact that the algebra is finite has the added advantage that it makes the mathematics very easy to manipulate. It also has the advantage that it is much easier to see exactly how a discrete space emerges from the algebra. It is then straight forward to see how the continuum emerges in the limit.

To show how it is possible to abstract a phase space from this algebra, we must first explain how it is possible to abstract a 'generalised point' from the algebra. An important clue as to how to proceed has been given by Eddington (1958), who argued that, within a purely algebraic approach to physical phenomena, there are elements of existence defined, not in terms of some metaphysical concept of existence, but in the sense that existence is represented by an algebraic element that contains only two possibilities: existence or non-existence. He assumed that existence could be represented by an idempotent element in an appropriate algebra. In our view it is not so much that points exist, but that they *persist*. They persist not in the sense that they are static, but rather that each point continually transforms into itself. Now the elements that transform into themselves are the idempotents, e, which are defined by the relation $e.e = e$. (Recall that in our approach a product represents succession.) Thus, in the algebra, generalised points correspond to a set of idempotents in the algebra.

Again since the Weyl algebra is finite, it is possible to find a complete set of pairwise orthogonal primitive idempotents e_i. One such set is

$$e_i = \frac{1}{n}\sum_k \omega^{-ik} e_0^k \tag{2}$$

The e_i will satisfy the relation $\sum_i e_i = 1$ and $e_i^2 = e_i$. The set e_i will constitute a set of generalised points in our phase space. To show that this set can be used to represent a finite set of points in 'space', let us introduce a 'position' operator X defined by

$$X = \frac{1}{n}\sum_{jk} j\omega^{-jk} e_1^0 = \sum_j j e^j \tag{3}$$

so that

$$Xe^j = je^j \tag{4}$$

Thus each primitive idempotent is labelled by the eigenvalue of the 'position' operator. If we choose e_0 as the point of the origin and e_1 as its neighbour then it can be shown that there exists a translation operator, T, such that

$$e_1 = Te_0T^{-1} \tag{5}$$

In this way we can generate all the points by successive application of T. Thus we generate all the points in the space from the algebra itself. Of course in this very simple model the space is only one dimensional, but the approach can be generalised to any dimension by using the full symplectic Clifford algebra.

It is perhaps worth mentioning here that this set of idempotents is not unique. It is always possible to find another set under a suitable inner automorphism of the algebra. Indeed there are pairs of spaces within the algebra that are related in such a way that one set can be chosen to be the position space while the dual pair will be the momentum space. Notice that what is called complementarity in standard quantum mechanics takes on an ontological significance in our approach in the sense that it is not possible to 'display' the position space at the same time as 'displaying' the momentum space. This is very reminiscent of David Bohm's notion of the explicate order (Bohm 1980). Recall that not all explicate orders can be made manifest together. So in our view the algebra is a description of the implicate order, with one explicate order representing the position space, while the momentum space is represented by another complementary explicate order. Thus within the algebra there is the possibility of finding many different generalised phase spaces.

In the finite algebra, all these spaces are equivalent, but when we go to an algebra with an infinite number of degrees of freedom, as is the case for quantum field theory, there emerges the possibility of many inequivalent spaces. (See Umewaza 1993). Within these we would expect the ultimate emergence of a preferred space-time will arise as a result of some as yet unknown symmetry breaking process.

The continuous phase space algebra can be generated by $[1, a, a^\dagger, \{\mathrm{P}\}]$ where a and $a^\dagger$ satisfy the commutation relation $[a, a^\dagger] = 1$. $\{\mathrm{P}\}$ is an idempotent that satisfies the relations

$\{\mathrm{P}\}^2 = \mathrm{P}$; $\quad a\{\mathrm{P}\} = 0$, $\quad$ and $\quad \{\mathrm{P}\}\, a^\dagger = 0$.

By using the translation operator which can be written in the form

$T(\alpha) = exp[\alpha^* a + \alpha a^\dagger]$ we can generate a continuum of points by the inner automorphism

$\{P_n\} = T(\alpha)^n \{P\} T(\alpha)^{-n}$

By increasing the number generators $a_j, a_j^\dagger$ we can construct higher dimensional spaces within which an algebraic geometry can be constructed from the elements of the algebra as shown in Fernandes (1996) and Fernandes and Hiley (1996). This structure has many similarities with the Einstein algebra introduced by Geroch (1972).

5 Process and Time

In the above discussions, we have been exploring algebras that are actually used in quantum mechanics. When it comes to time, we cannot use these methods because nowhere does time appear as an element of an operator algebra in quantum mechanics. Thus it is not possible simply to exploit the mathematics as we have done in the previous section. The problem of finding a suitable time operator is by now well documented and various possibilities of finding a suitable operator within more general schemes have been proposed (Unhruh and Wald 1989 and Isham 1991). Of the various attempts that have been made, the most well developed are the proposals of Prigogine and his group (see Petrosky and Prigogine 1990, 1994). Our approach shares some of the techniques used by this group although their work is far more extensive.

There is another interesting approach emerging from the work of Umazawa (1993) who has been exploring an algebraic approach based on thermal field theory. Within this approach a specific time operator has been proposed by Ban (1991) and it is this work that is most closely related to the ideas that we will now discuss. Ban's approach enables us to develop further the ideas on time proposed by Bohm (1986). It is this extension that we now want to address here.

One of the key new ideas described in the previous section was the way it is possible to abstract the notion of a 'point' from the algebra itself. This offered a way to build a space from the algebra itself. When considering the question of time, it is tempting to try to exploit a similar idea, namely, to attempt to construct the notion of an instant of time from some suitable algebra, but we have two problems. Firstly, as we have already remarked, we have no suitable algebra containing an element which can be identified with thme. Secondly the notion of an 'instant' of time cannot be regarded as a 'point' that persists. Time is about becoming, not persisting. In our approach we are attempting to find time in nature so that it emerges from process. We are not trying to explain the development of nature in time. In the latter, time appears as a "metaphysical enigma" without any verifiable content. As Whitehead (1930) puts it:

> There is time because there are happenings, and apart from these happenings there is nothing

Here a 'happening' is not sharply defined. It is more like a moment which has an inner structure of its own. Each moment contains both the memories of the past with the potentialities for future development.

We can obtain a better feeling for the notion of a moment by examining our own perception of time. Any process of which we become aware has already past, even if it is only for a fraction of a second. What we are conscious of is a memory of the past tinged with an expectation of the future. In the words of Bohm (1986):

> Although the present is, it cannot be specified in words or thoughts without slipping into the past.

Further recall that in order to perceive objects in motion, we need to integrate the information that reaches the retina over about a fifth of a second. For example, slow down the speed at which a cinefilm is run and the image of the motion becomes a series of discontinuous jumps. Speed the film up and we perceive a smooth unfolding motion as normally experienced. The brain has merged the images to produce not only the visual experience of motion, but it can also produce the physical experience of motion if the conditions are right. Thus time is experienced not as an instant, but as an extended moment, a notion that is of necessity ambiguous.

This ambiguity does not only arise from philosophical and psychological considerations. It also arises directly from quantum physics itself. Firstly the ambiguous nature of a moment receives support through the energy-time uncertainty relationship. Recall Einstein's gedanken experiment of determining the time at which a photon leaves a box. The time this occurs is determined by observing when the box changes its weight. To determine this weight change, the box is suspended by a spring fixed to a rigid external frame. However because of the position-momentum uncertainty, there is an ambiguity in the position of the box relative to the fixed frame and thus the moment the photon leaves the box is ambiguous. This means that the timing of any change that occurs in the box is ambiguous relative to the timing of changes occurring in the fixed frame.

There is a different but related ambiguity that has been pointed out by Peres (1980) who discusses the problems involved in constructing a quantum clock. One of the significant conclusions in his paper is that nonlocality in time is an essential feature of all quantum clocks. He goes on to conclude

> It seems that the Schrödinger wave function $\psi(t)$ with its continuous time evolution given by $i\hbar\dot{\psi} = H\psi$, is an idealisation

> rooted in classical theory. It is operationally ill defined (except in the limiting case of stationary states) and should probably give way to a more complicated dynamical formalism, perhaps one nonlocal in time. Thus in retrospect, the Hamiltonian approach to quantum physics carries the seeds of its own demise.

These considerations suggest that we must give up the idea of trying to describe motion as a precise point-to-point development in time. We propose that an appropriate description will entail integrating over what would, in the usual approach, be regarded as a finite period of external time.

Bohm and Hiley (1981) have already provided some support for these proposals. We started by examining the differences between the way classical and quantum phase space motion is generated. In contrast to the classical picture where the motion involves a point-to-point substitution through the group of canonical transformations, quantum processes require that the contribution to a point on the quantum motion involves an integration over an extended range of points in a generalised phase space.

To arrive at this conclusion it is necessary to assume that it is the density matrix rather than the wave function that provides the most complete description of an evolving quantum process. A similar assumption is introduced by Prigogine (1993) in his work on irreversible quantum processes. To generalise the dynamics further, we replace the density matrix $\rho(x, x')$ by a non-Hermitean matrix which we called the characteristic matrix $\xi(x, x')$. The characteristic matrix is then used to provide a description of the evolution of the generalised quantum state. We began by assuming that the characteristic matrix satisfies the Liouville equation

$$i\frac{\partial \xi}{\partial t} = H\xi - \xi H = [H(x,p) - H^*(x',p')]\xi = \hat{L}\xi \qquad (6)$$

where $H(x,p)$ is the Hamiltonian of the system and $\hat{L}$ is the Liouville superoperator given by

$$\hat{L} = H \otimes 1 - 1 \otimes H. \qquad (7)$$

The last step in equation (6) is achieved by writing the characteristic square matrix as a column matrix. This means that we are working with a superalgebra where $\hat{L}$ takes the form given by equation (7). In order to proceed it is necessary to change co-ordinates to $X = (x + x')/2$, and $\eta = x - x'$, and then introduce the Wigner-Moyal transformation

$$F(X, P) = \int U(P, \eta)\xi(X, \eta)d\eta, \qquad (8)$$

where

$$U(P,\eta)=\frac{1}{2\pi}exp[iP\eta], \tag{9}$$

with $P=(p+p')/2$. The Liouville equation then becomes

$$\frac{\partial F(X,P)}{\partial t}+\frac{P}{m}\frac{\partial F(X,P)}{\partial X}+\int L(P,P')F(X,P')dP'=0 \tag{10}$$

where

$$L(P,P')=iV_{2(P-P')}exp[2(P-P')X]-V_{2(P'-P)}exp[2(P'-P)X] \tag{11}$$

$V_{2(P-P')}$ being the Fourier component of the potential. $F(X,P)$ is a complex function which is analogous to the classical quasi-probability density used in the Wigner-Moyal interpretation of phase space, but we do not use it as a probability density.

In Bohm and Hiley (1981) a positive definite probability is obtained by constructing a statistical matrix $w(x,x')$ from the characteristic matrix

$$\xi(x,x')=\sum_i\sqrt{p_i}e^{i\phi}\psi_i^*(x')\psi_i(x). \tag{12}$$

so that

$w(x,x')=\int\tilde{\xi}(x',x'')\xi(x'',x)dx''$

Here $\tilde{\xi}=(\xi^*)^T$ with T being the transpose. Thus the characteristic matrix is a kind of "square-root" of the density matrix. The details of this approach to the classical limit will not concern us further in this paper.

What we wish to point out here is that the generalised motion requires *two* points in configuration space rather than one as used in classical physics. In other words we need both phase space points (x,p) and (x',p'). Then in discussing a quantum process, (x,p) and (x',p') become operators so that in algebraic terms this means we have a pair of symplectic algebras, one attached to each point of phase space. We then assume this pair of algebras form a superalgebra with

$$\begin{aligned} X &= (x\otimes 1+1\otimes x')/2 \qquad \eta=x\otimes 1-1\otimes x' \\ P &= (p\otimes 1+1\otimes p')/2 \qquad \pi=p\otimes 1-1\otimes p' \end{aligned} \tag{13}$$

which give the commutation relations

$$[X,P]=[\eta,\pi]=[X,\eta]=[P,\pi]=0; \qquad [X,\pi]=[\eta,P]=i \tag{14}$$

Furthermore if we follow through the ideas outlined in the previous section, we can now abstract a phase space from the underlying process where neighbouring points are related, not by externally imposed neighbourhood relations, but by relationships provided by the generalised dynamics itself. Thus the structure of the abstracted phase space is generated by the underlying quantum processes themselves.

Recalling that a symplectic algebra can be generated by the Fock boson algebra, our structure thus contains two Fock algebras. The elements (x, p) and (x', p') are then replaced by pairs of elements$(a, a^\dagger)$ and $(\tilde{a}, \tilde{a}^\dagger)$, with
$a = x + ip, \quad a^\dagger = x - ip, \qquad \text{and} \qquad \tilde{a} = x' + ip', \quad \tilde{a}^\dagger = x' - ip'.$

The algebraic structure that we have introduced is now identical to the structure introduced by Takahashi and Umezawa (1975) and forms the basis of what they have called thermo field dynamics (See Umewaza 1993). In their original work, because the emphasis was placed on the field theoretic meaning of a and $a^\dagger$, (i.e., that they are particle annihilation and creation operators) Takahashi and Umezawa were forced to regard the additional pair of operators as arising from a 'fictitious system', a notion that does not encourage confidence. However from the point of view that we are adopting here, it appears as a natural consequence of the way we must describe the evolution of quantum systems in a phase space. Furthermore it provides the means of investigating the possible connections between thermal physics and quantum mechanics at a deep level. Before proceeding along these lines we need to generalise the notion of time.

To begin the discussion on time, it should first note that in the evolution described by the Schrödinger equation, and, in consequence, in the Liouville equation, nothing ever actually happens. By this we mean that the unitary evolution is simply a re-description of the process. This redescription is parameterised by an external variable, t, which we can correlate to our classical clocks and therefore we call the parameter 'time'. Now to make something happen within quantum theory, we need a measurement in order to produce an actual result. In an ontological description this means we need some non-unitary transformation to actualise the process. A description of this feature is something that the standard interpretation does not provide.

Let us proceed naively and argue that we need to consider a two-time characteristic matrix $\xi(x, x'; t, t')$, the two times enabling us to incorporate the notion of a moment which, in the first approximation, is characterised by a mean time $T = (t+t')/2$ and a time difference $\tau = t - t'$. Since there is an ambiguity in the relationship between time and energy, we must necessarily have an ambiguity in the energy of the system. We can characterise this ambiguity by a difference in energy $\varepsilon = E' - E$, about a mean energy $\bar{E} = (E' + E)/2$. In order to extend these ideas to an algebraic theory, we use the analogy with the commutation relations (14) derived above and we define a set of commutation relations for $T, \tau, \bar{E}$ and ε as follows:

$$[T, \bar{E}] = [\tau, \varepsilon] = [T, \tau] = [\bar{E}, \varepsilon] = 0 \qquad [T, \varepsilon] = [\tau, \bar{E}] = i \tag{15}$$

Thus if we treat the operator T as a 'time', then the commutation relation $[T, \varepsilon] = i$ is identical to the time operator introduced by Ban (1991). This gives an ambiguity in specifying the mean time of a process passing between two energies. Unfortunately the second commutator in (15) can only be regarded as an approximation because $\bar{E}$ is bounded from below. Nevertheless this commutation relation can be regarded as indicating that there is some ambiguity in how long a system can remain in a state with a given spread of its mean energy and this is clearly related to the life-time of the process.

In requiring a description which needs a pair of algebras to capture the structure of quantum processes we are essentially constructing a bi-algebra, the superalgebra, and our proposal is that this structure is the appropriate mathematical descriptive form that we need to describe a process based physics. This means that we can write $\varepsilon = H \otimes 1 - 1 \otimes H$ which we immediately recognise it as the Liouville operator introduced above. Thus we can also write the commutator in the form $[T, \hat{L}] = i$, which is essentially the same relation introduced by Prigogine (1980). In Prigogine's approach T was regarded as the 'age' of the process. We prefer to call it internal time.

If we regard L as generating the evolution of the process, then the Heisenberg equation of motion for the operator T is

$$\frac{dT}{dt} = i[\hat{L}, T] = 1 \tag{16}$$

which has the solution $T = t + t_0$. This means that the internal time increases linearly with external time. We now have the possibility of developing an algebraic theory of time but this will require us to consider more carefully the structure of the bi-algebra and to develop a connection between time and entropy.

To begin a discussion of this relationship let us first see in what sense a moment can be described by a two-time density matrix. Consider two wave functions $\Psi_1(x, t)$ and $\Psi_2(x', t')$. After expanding these wave functions in terms of a complete set of energy eigenfunctions, we can form the density matrix

$$\rho_{EE'}(x,x',t,t') = \Psi^*_{E'}(x')\Psi_E(x)exp[-i(Et - E't')]/\hbar \tag{17}$$

If we now change the time co-ordinates to T and τ, we find

$$\rho_{EE'}(x,x',T,\tau) = \Psi^*_{E'}(x')\Psi_E(x)exp[-i(\varepsilon T + \bar{E}\tau)]/\hbar \tag{18}$$

Thus the density matrix is characterised by a mean time T and a time difference τ which, as we have already suggested, should be regarded as the time spent passing between the two energy states.

Now let us turn to thermal field theory. Here the inner automorphism

$$U(\beta) = exp[\beta(a\tilde{a} - \tilde{a}^\dagger a^\dagger)] \tag{19}$$

produces a representation of the algebra in which the new vacuum state that can be characterised by a temperature $\theta = 1/k\beta$. This vacuum state is represented by a superwave function from which we can project an ordinary wave function which can be written in the form:

$$\Psi(x,t) = \sum_E exp[-\beta E/2]\Psi_E(x)exp[-iEt/\hbar] \tag{20}$$

We can interpret this wave function by assuming that there is an underlying process which generates a movement that is equivalent to a thermally induced diffusion, so that we can charaterise this movement with an equivalent temperature θ. The corresponding density matrix constructed from a pair of wave functions of the type (20) is

$$\rho_{EE'}(x,x',T,\beta) = \sum_{EE'} exp[-\beta E/2]\Psi^*_{E'}(x')\Psi_E(x)exp[-i\varepsilon T/\hbar] \tag{21}$$

By comparing the two expressions (18) and (21) for $\rho_{EE'}$, we see that τ can be identified with $\hbar\beta/2$. In other words the time between states can be characterised by the temperature θ . Now let us consider the correlation function $< t+\tau, t >$ defined by

$$< t+\tau, t > = \int \Psi^*(x,t+\tau)\Psi(x,t)dx$$

$$= \int \sum_{EE'} exp[-\beta(E+E')/2]\Psi^*_{E'}(x')\Psi_E(x)exp[i(E(t+\tau)/2 - Et]/\hbar dx \tag{22}$$

Using the orthogonality of the set $\Psi_E(x)$, this expression reduces to

$$< t+\tau, t >= \sum_E exp[-\beta E] exp[-iE\tau/\hbar] \tag{23}$$

Therefore in a time characterised by $\tau > \hbar\beta$ the correlation $< t+\tau, t > \rightarrow 0$. What this implies is that as the diffusive process proceeds, successive wave functions become orthogonal. The superwave functions from which the wave function (20) is derived can be characterised by its entropy, thus wave functions which differ significantly in entropy will be orthogonal. Therefore if each moment of time is characterised by one of these wave functions we can begin to understand how successive moments and the increase of entropy become related.

For a small enough entropy change there is a direct connection from one moment to the next. However when the entropy difference becomes large enough, then the moments are independent of each other. It was this orthogonality that allowed Bohm (1986) to argue that in the implicate order all moments of time were present together in what can be regarded as a timeless order. In spite of this, the order of time could unfold through a series of explicate orders. Thus what seemed like a vague general notion can now be given a more explicit mathematical form. In our view, this mathematical structure contains sufficient structure to enable us to provide an algebraic description of process from which space-time can be abstracted. It is hoped that this new approach will provide some insight into how gravity can be encompassed as a quantum phenomenon.

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